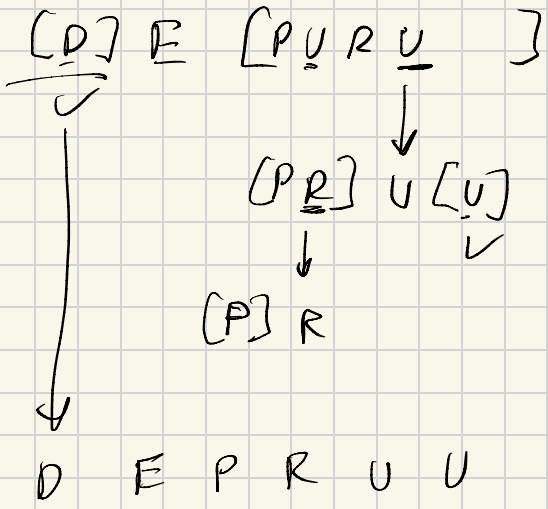
PSO7

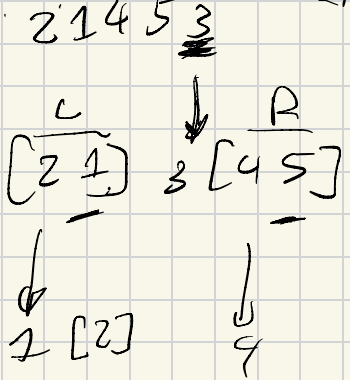
Quadratic Hashing, Union Find, BST

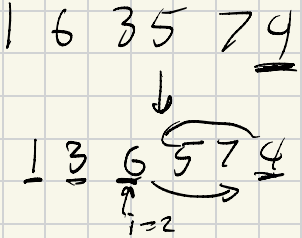
Midterm:

P U R D U E




```
quicksort(A):
  p = A[-1]
(L,R) = Partition (A, p)
quicksort(L) @ [p] @ quicksort(R)
```





You are in the role of a hacker trying to break down a hash table. The information collected so far indicates the hash table uses Quadratic Probing with $h(k,i) = (k+i^2) \bmod m$ for collision management and its current capacity is $m = 9$. The current state of the table is:

0	1	2	3	4	5	6	7	8
17	28	20			5	32		19

The system is nearly overloaded and will collapse if the next item inserted causes at least 4 probes. As an attacker you are considering inserting the following keys: 16, 35 and 10. Which (if any) of these values would bring the system down if inserted next? Explain your answer.

another quadratic hash
$h(k,i) = k + i + i^2 \bmod m$
quadratic

h(key, iteration)

Quadratic probing:

i = i'th collision

first attempt: h(key, 0) [collision]
h(key, 1)
:

Trying 16

0	1	2	3	4	5	6	7	8
17	28	20			5	32		19

16

$h(16,0) = 16 + 0^2 \bmod 9 = 7$

No collision

Trying 35

0	1	2	3	4	5	6	7	8
17	28	20			5	32		19

$h(35,0) = 35 + 0^2 \bmod 9 =$ 8

Trying 35

0	1	2	3	4	5	6	7	8
17	28	20			5	32		19

h(35,0) = 35 + 0^2 mod 9 = 8 **Collision**

h(35,1) = 35 + 1^2 mod 9 = 0

Trying 35

0	1	2	3	4	5	6	7	8
17	28	20			5	32		19

h(35,0) = 35 + 0^2 mod 9 = 8 **Collision**

h(35,1) = 35 + 1^2 mod 9 = 0 **Collision**

h(35,2) = 35 + 2^2 mod 9 = 3

Trying 35

0	1	2	3	4	5	6	7	8
17	28	20			5	32		19

h(35,0) = 35 + 0^2 mod 9 = 8 **Collision**

h(35,1) = 35 + 1^2 mod 9 = 0 **Collision**

h(35,2) = 35 + 2^2 mod 9 = 3 **No Collision**

2 collisions

Trying 10

0	1	2	3	4	5	6	7	8
17	28	20			5	32		19

$h(10,0) = 10 + 0^2 \bmod 9 =$

Trying 10

0	1	2	3	4	5	6	7	8
17	28	20			5	32		19

h(10,0) = 10 + 0^2 mod 9 = 1 **Collision**

h(10,1) = 10 + 1^2 mod 9 =

Trying 10

0	1	2	3	4	5	6	7	8
17	28	20			5	32		19

h(10,0) = 10 + 0^2 mod 9 = 1 **Collision**

h(10,1) = 10 + 1^2 mod 9 = 2 **Collision**

h(10,2) = 10 + 2^2 mod 9 =

Trying 10

0	1	2	3	4	5	6	7	8
17	28	20			5	32		19

h(10,0) = 10 + 0^2 mod 9 = 1 **Collision**

h(10,1) = 10 + 1^2 mod 9 = 2 **Collision**

h(10,2) = 10 + 2^2 mod 9 = 5 **Collision**

h(10,3) = 10 + 3^2 mod 9 =

Trying 10

0	1	2	3	4	5	6	7	8
17	28	20			5	32		19

h(10,0) = 10 + 0^2 mod 9 = 1 **Collision**

h(10,1) = 10 + 1^2 mod 9 = 2 **Collision**

h(10,2) = 10 + 2^2 mod 9 = 5 **Collision**

h(10,3) = 10 + 3^2 mod 9 = 1 **Collision**

10 gives us a collision.

Question 2

(1) What is the asymptotic performance of inserting n items with keys sorted in a descending order into an initially empty binary search tree?

(2) Is the operation of deletion "commutative" in the sense that deleting x and then y from a binary search tree leaves the same tree as deleting y and then x? Argue why it is or give a counterexample.

(3) Your friend thinks he has discovered a remarkable property of binary search trees. Suppose that the search for key k in a binary search tree ends up in a leaf. Consider three sets: A, the keys to the left of the search path; B, the keys on the search path; and C, the keys to the right of the search path. Your friend claims that any three keys $a \in A$, $b \in B$, and $c \in C$ must satisfy $a \leq b \leq c$. Give a simple counterexample to his claim.

Insert(root,x):

If root == null: return x

If (x <= root.val): insert(root.left,x)

If (x > root.val): insert(root.right,x)

https://justin-zhang.com/teaching/cs251Old/S25/pso6Noted.pdf

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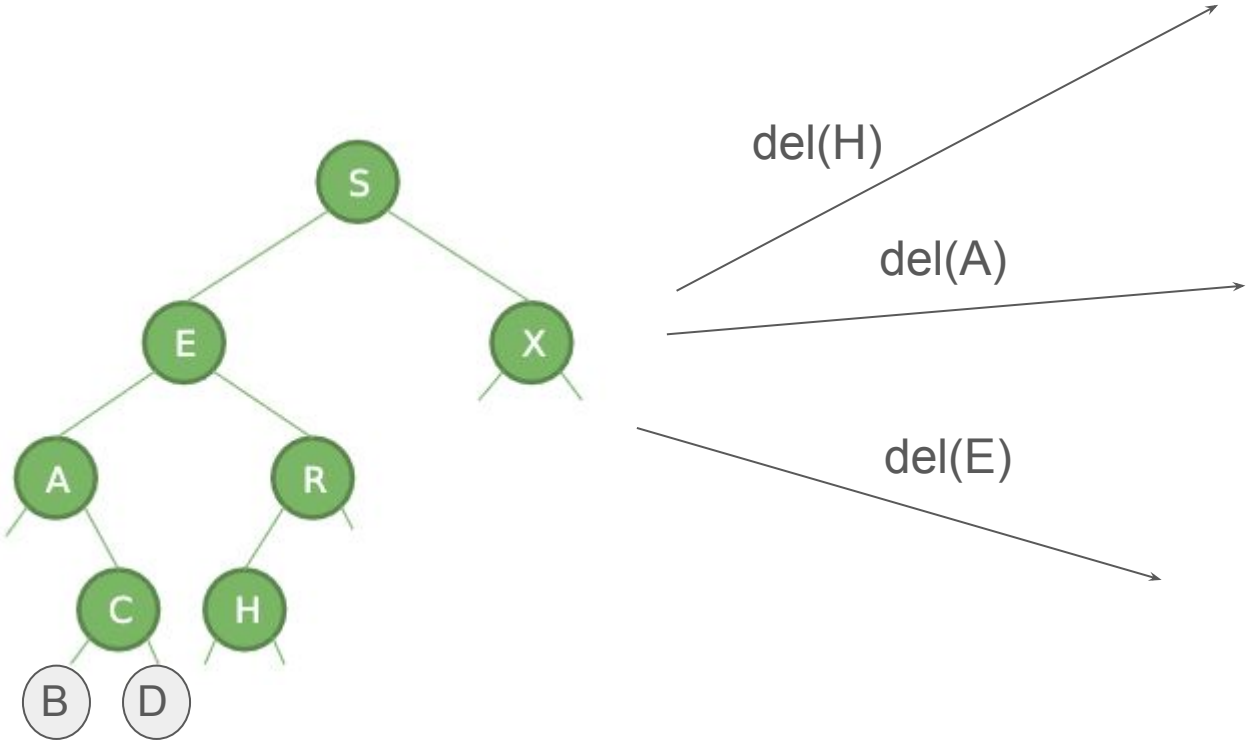
How does deletion work?

Deletion in a BST: Depends on # children

Basically, want to delete while keeping order



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Assume 1 child deletion swaps with **successor**

Delete A, then B	
Delete B, then A	

Question 2

(1) What is the asymptotic performance of inserting n items with keys sorted in a descending order into an initially empty binary search tree?

(2) Is the operation of deletion "commutative" in the sense that deleting x and then y from a binary search tree leaves the same tree as deleting y and then x? Argue why it is or give a counterexample.

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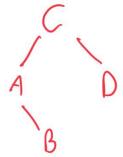
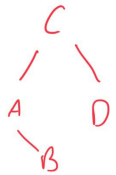
If 1 child deletion swaps with **predecessor**

delete C, then delete D



delete D, then delete C



Question 2

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(Union find)

1. Suppose that we implemented the Union-Find data structure with **quick-find**. The current state of the data-structure is defined in the following table.

i	0	1	2	3	4	5	6	7	8	9
Id[i]	1	1	7	3	3	3	7	7	1	1

List each disjoint set.

Union find
- be able to check set membership (find)
- be able to do unions

S_1 S_2 S_3
$\{a,b\}$ $\{c\}$ $\{d\}$

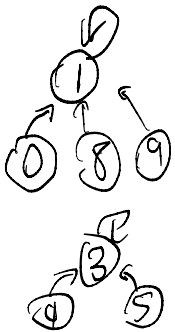
is $a \in S_1$?

What is Quick Find?

optimizes

quick find

id[i] ≅ i is in set id[i]



$\{0,1,8,9\}$ $\{2,6,7\}$ $\{3,4,5\}$
S_1 S_7 S_3

Union(1,2) → $S_4 = \{a,b,c\}$
$= S_1 \cup S_2$.

find(5) = S_3

```
def find(e):
    return id[e]  //O(1)
```

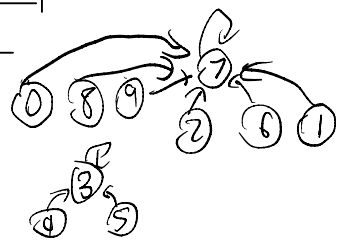
2. What does the table of the union-find data structure look like after running the following two unions: `Union`$(5,4)$, `Union`$(0,7)$?

$\{0,1,8,9\}$ $\{2,6,7\}$ $\{3,4,5\}$

i	0	1	2	3	4	5	6	7	8	9
Id[i]	1	1	7	3	3	3	7	7	1	1

`Union(5,4)` – both in S_3 so table unchanged.

i	0	1	2	3	4	5	6	7	8	9
id[i]	1	1	7	3	3	3	7	7	1	1



`Union(0,7)`

Union(a,b): $O(n)$ time.
- go through all elts pointing to parent(a); make them point to parent(b).

i	0	1	2	3	4	5	6	7	8	9
id[i]	7	7	7	3	3	3	7	7	7	7

Question 3

(Union find)

1. Suppose that we implemented Union-Find data structure with quick-union. The current state of the data-structure is defined in the following table.

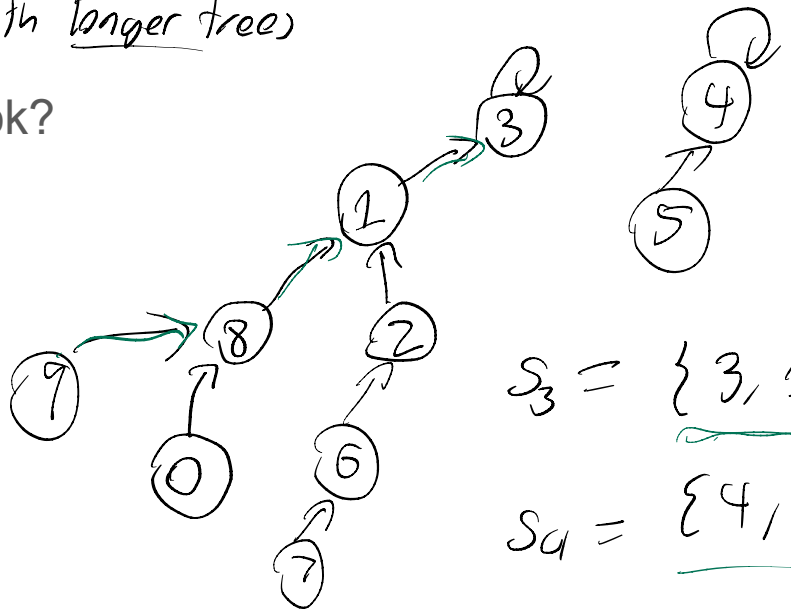
i	0	1	2	3	4	5	6	7	8	9
Id[i]	8	3	1	3	4	4	2	6	1	8

List each disjoint set along with its canonical element (Hint: It may help to draw the corresponding trees).

What is quick union?

- quickfind with longer trees

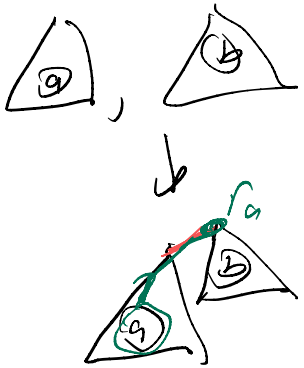
How do the trees look?



ex: find(9)
= 3

// O(longest path in tree)

union(a,b)



find(a) = r_a

$S_3 = \{3, 1, 8, 9, 0, 2, 6, 7\}$

$S_4 = \{4, 5\}$

i	0	1	2	3	4	5	6	7	8	9
Id[i]	8	3	1	3	4	4	2	6	1	8

2. Suppose we optimize our construction by implementing path compression and union-by-weight. We then run Union(6, 5). What is updated state of the Union–Find data structure? (Note: Refer to the table in part (a) for the initial state of the union-find data structure.)

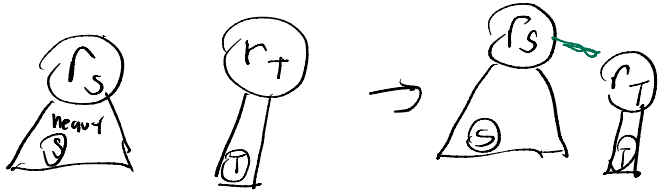
Path compression:

Anytime I traverse the tree (find/union), I move all traversed nodes up to the root

ex: find(4) = 1

Union by weight:

Union(S,T)



Put heavier tree as the root

i	0	1	2	3	4	5	6	7	8	9
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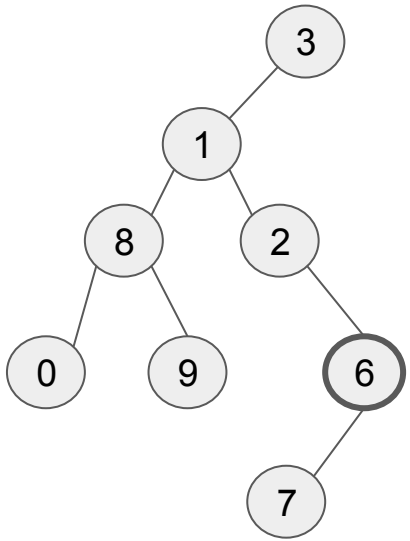
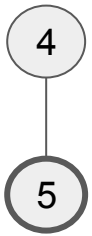
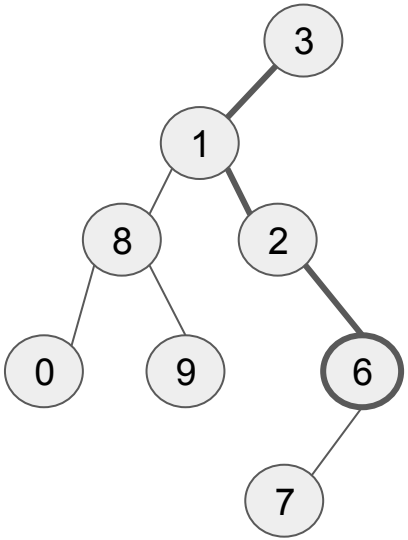




Union(5,6)

i	0	1	2	3	4	5	6	7	8	9
Id[i]	8	3	1	3	4	4	2	6	1	8

2. Suppose we optimize our construction by implementing path compression and union-by-weight. We then run Union(6, 5). What is updated state of the Union–Find data structure? (Note: Refer to the table in part (a) for the initial state of the union-find data structure.)



Union(5,6)
Step 1: find their roots by traversing up the tree
-Path compress

i	0	1	2	3	4	5	6	7	8	9
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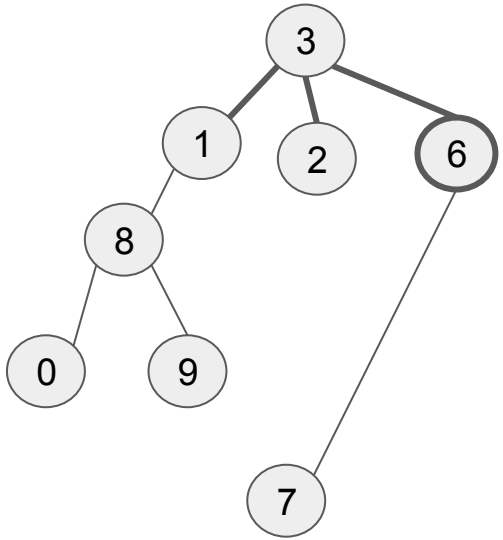

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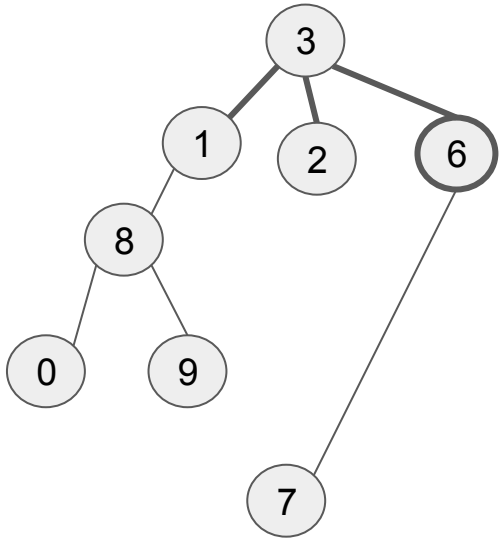


union(a,b):
r_a = find(a)
r_b = find(b)
union-by-weight(r_a, r_b)

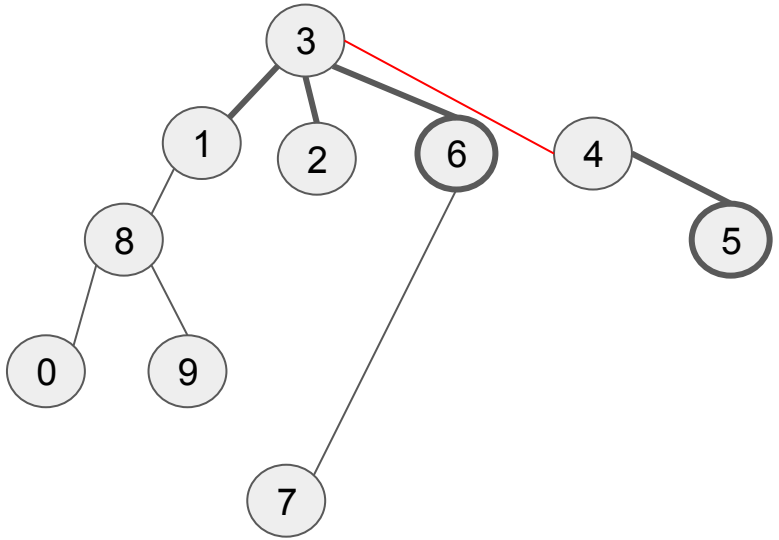
find(a): // path comp.

Union(5,6)
Step 1: find their roots by traversing up the tree
Path compress

Step 2: connect roots
Union by weight
(minimize suffering!)

i	0	1	2	3	4	5	6	7	8	9
Id[i]	8	3	1	3	4	4	2	6	1	8

2. Suppose we optimize our construction by implementing path compression and union-by-weight. We then run Union(6, 5). What is updated state of the Union–Find data structure? (Note: Refer to the table in part (a) for the initial state of the union-find data structure.)



Union(5,6)
Step 1: find their roots by traversing up the tree
Path compress

Step 2: connect roots
Union by weight
(minimize suffering!)

i	0	1	2	3	4	5	6	7	8	9
Id[i]	8	3	1	3	4	4	2	6	1	8

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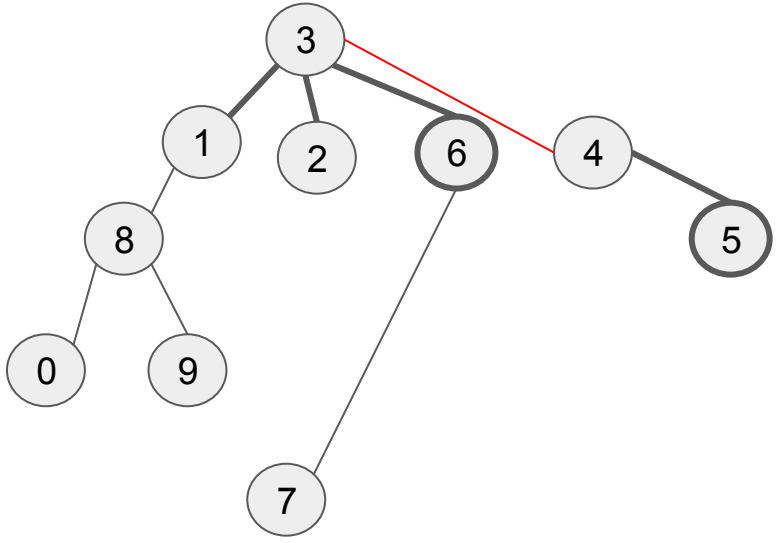

Union(5,6)
Step 1: find their roots by traversing up the tree
Path compress

Step 2: connect roots
Union by weight
(minimize suffering!)
Step 3: Update the table

i	0	1	2	3	4	5	6	7	8	9
Id[i]	8	3	3	3	3	4	3	6	1	8