PSO 10

Strong and Weak Connection

https://justin-zhang.com/teaching/CS251

Question 1

(Articulation point)

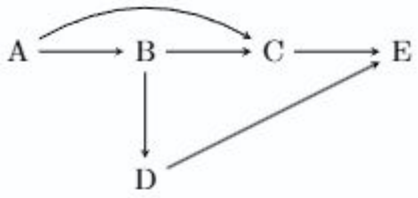
We define an *articulation point* as a vertex that when removed causes a connected graph to become disconnected. For this problem, we will try to find the articulation points in an undirected graph G.

(1) How can we efficiently check whether or not a graph is disconnected?

(2) How to determine if a node u is an articulation point or not?

Question 2

Consider the directed graph $G = (V, E)$ given below:



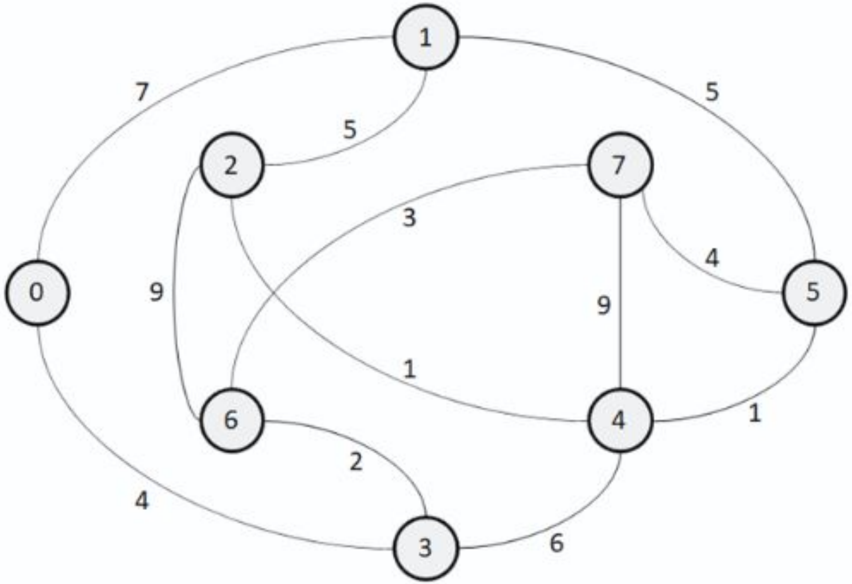
where the set of vertices is $V = \{A, B, C, D, E\}$ and the set of edges is:

$$E = \{(A, B), (A, C), (B, C), (B, D), (C, E), (D, E)\}.$$

1. Construct the adjacency matrix A of G.
2. Compute the transitive closure of G using Warshall's algorithm.
3. Draw the graph representation of the transitive closure of G.
4. Determine the reachability of each node in G.
5. Identify if G is strongly connected. If not, can you add one edge to make G become a strongly connected graph?

Question 3

Consider the following graph G:



Let G_d be a directed graph using the vertices of G. For a pair of vertices u and v connected by an edge in G, their respective directed edge in G_d is as follows:

$$\text{Edge with vertices } u \text{ and } v = \begin{cases} (u, v), & \deg(u) < \deg(v) \vee (\deg(u) = \deg(v) \wedge u < v) \\ (v, u), & \text{Otherwise} \end{cases}$$

1. Is G_d strongly connected? If yes, explain why. Otherwise, list the minimum number of edges required to make G_d strongly connected.
2. Show all the topological orderings of G_d.

(undirected)

(1) How can we efficiently check whether or not a graph is disconnected?

Two vertices u,v are connected if there is some way to get from u → v.

Graph is connected if for *all* u,v vertices, u and v are connected

Are there any algorithms that can help us here?

(undirected)
(1) How can we efficiently check whether or not a graph is disconnected?

Two vertices u,v are connected if there is some way to get from u → v.

Graph is connected if for *all* u,v vertices, u and v are connected

Use BFS/DFS, count the number of vertices visited

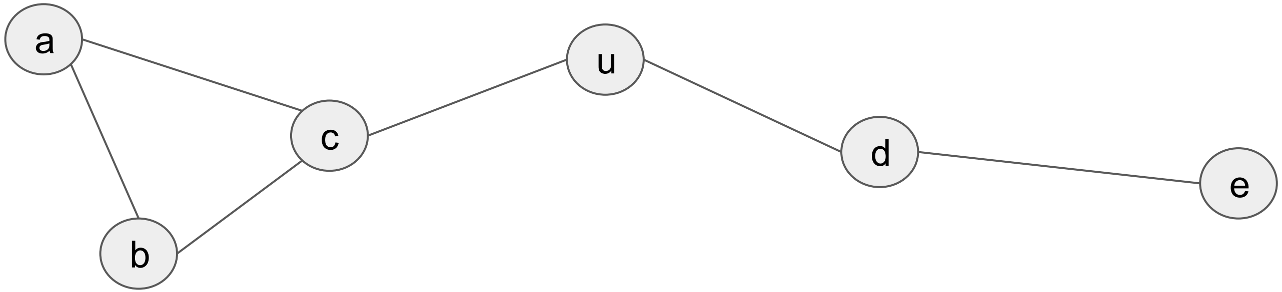
Question 1

(Articulation point)

We define an *articulation point* as a vertex that when removed causes a connected graph to become disconnected. For this problem, we will try to find the articulation points in an undirected graph G.

(2) How to determine if a node u is an articulation point or not?

Any idea from pt 1?



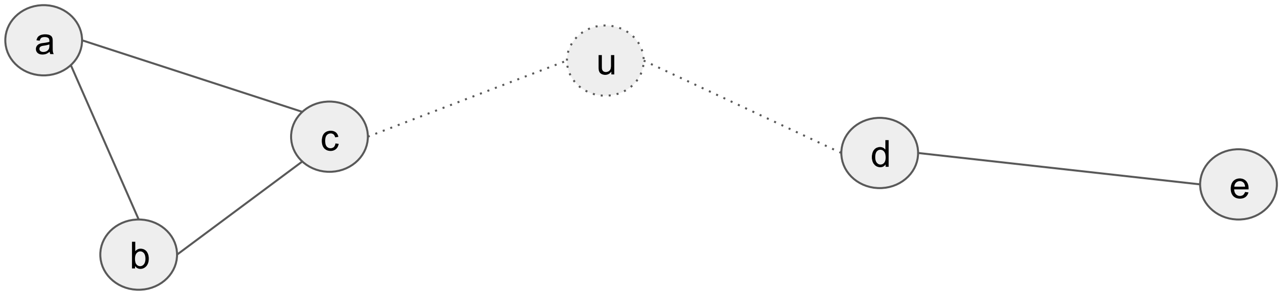
Question 1

(Articulation point)

We define an *articulation point* as a vertex that when removed causes a connected graph to become disconnected. For this problem, we will try to find the articulation points in an undirected graph G.

(2) How to determine if a node u is an articulation point or not?

Any idea from pt 1? Check connectivity when u is deleted

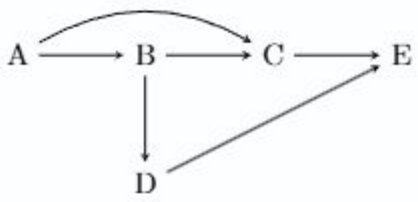


Exercise: Suppose you wanted to find all articulation points. Can you do so in O(|V| + |E|) time?

Hint: a point is an articulation point iff it is not in a cycle.

Question 2

Consider the directed graph $G = (V, E)$ given below:

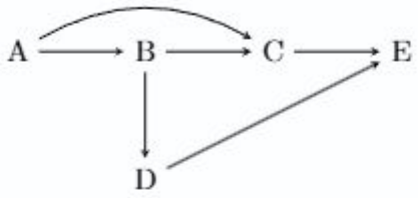


1. Construct the adjacency matrix A of G.

u/v	A	B	C	D	E
A	0				
B		0			
C			0		
D				0	
E					0

Question 2

Consider the directed graph $G = (V, E)$ given below:



2. Compute the transitive closure of G using Warshall's algorithm.

What's your algorithm Warshall/Floyd/Ingerman/Roy/Kleene?

History and naming [edit]

Worst-case space complexity	$\Theta(\|V\|^2)$

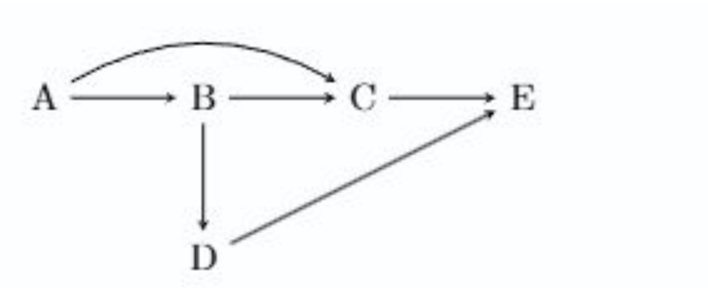
The Floyd–Warshall algorithm is an example of dynamic programming, and was published in its currently recognized form by Robert Floyd in 1962.[3] However, it is essentially the same as algorithms previously published by Bernard Roy in 1959[4] and also by Stephen Warshall in 1962[5] for finding the transitive closure of a graph,[6] and is closely related to Kleene's algorithm (published in 1956) for converting a deterministic finite automaton into a regular expression, with the difference being the use of a min-plus semiring.[7] The modern formulation of the algorithm as three nested for-loops was first described by Peter Ingerman, also in 1962.[8]

```
algorithm Floyd-Warshall(M:adjacency matrix representing G(V,E))

    R^(-1) ← M
    n ← |V|

    for k from 0 to n−1 do
        for i from 0 to n−1 do
            for j from 0 to n−1 do
                R^(k)[i,j] ← R^(k−1)[i,j] or (R^(k−1)[i,k] and R^(k−1)[k,j])
            end for
        end for
    end for

    return R^(n−1)
end algorithm
```



$R^{(-1)} \leftarrow M$

$n \leftarrow |V|$

for k from 0 to $n-1$ **do**

  for i from 0 to $n-1$ **do**

    for j from 0 to $n-1$ **do**

      $R^{(k)}[i,j] \leftarrow R^{(k-1)}[i,j]$ or $(R^{(k-1)}[i,k]$ and $R^{(k-1)}[k,j])$

    end for

  end for

end for

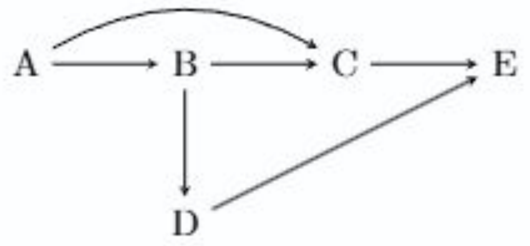
return $R^{(n-1)}$

end algorithm

$R^{(-1)}$	A	B	C	D	E
A	0	1	1	0	0
B	0	0	1	1	0
C	0	0	0	0	1
D	0	0	0	0	1
E	0	0	0	0	0

k = A

$R^{(0)}$	A	B	C	D	E
A					
B					
C					
D					
E					

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  R^(-1) ← M
  n ← |V|

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        end for
     end for
  end for

  return R^(n−1)
end algorithm
```

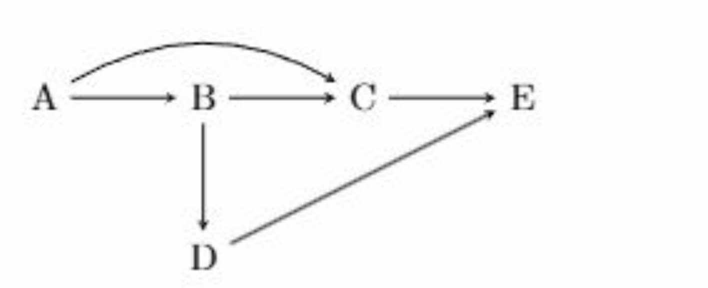
$R^{(-1)}$	A	B	C	D	E
A	0	1	1	0	0
B	0	0	1	1	0
C	0	0	0	0	1
D	0	0	0	0	1
E	0	0	0	0	0

k = A

→

i = A
j = A

$R^{(0)}$	A	B	C	D	E
A					
B					
C					
D					
E					

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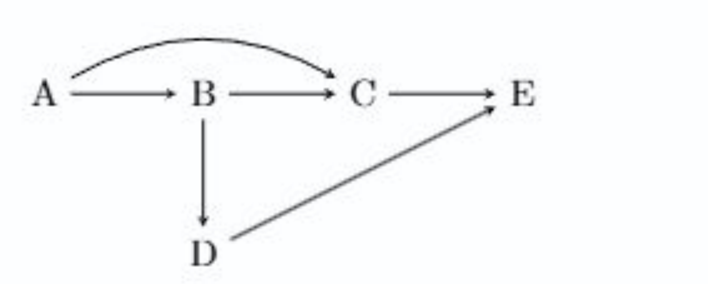
$R^{(-1)}$	A	B	C	D	E
A	0	1	1	0	0
B	0	0	1	1	0
C	0	0	0	0	1
D	0	0	0	0	1
E	0	0	0	0	0

k = A

i = A
j = A

$R^{(0)}$	A	B	C	D	E
A	0				
B					
C					
D					
E					

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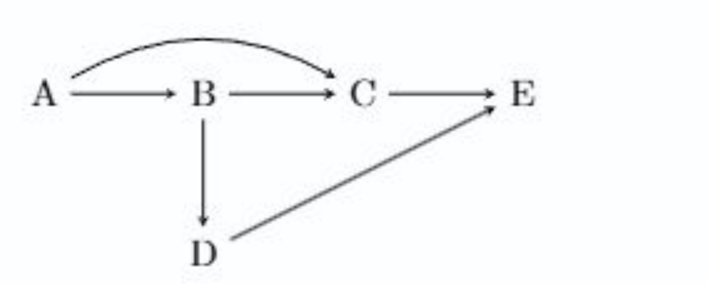
$R^{(-1)}$	A	B	C	D	E
A	0	1	1	0	0
B	0	0	1	1	0
C	0	0	0	0	1
D	0	0	0	0	1
E	0	0	0	0	0

k = A

→

i = A
j = B

$R^{(0)}$	A	B	C	D	E
A	0	1			
B					
C					
D					
E					

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end algorithm
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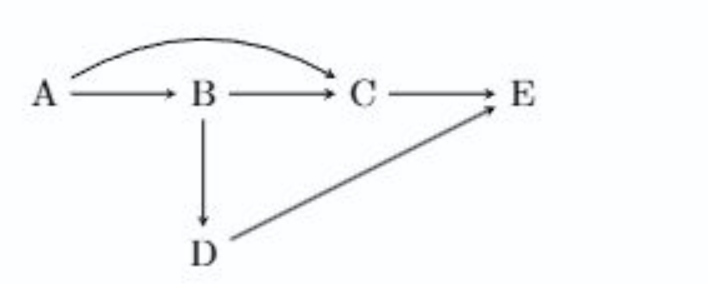
$R^{(-1)}$	A	B	C	D	E
A	0	1	1	0	0
B	0	0	1	1	0
C	0	0	0	0	1
D	0	0	0	0	1
E	0	0	0	0	0

k = A

i = A
j = C

$R^{(0)}$	A	B	C	D	E
A	0	1			
B					
C					
D					
E					



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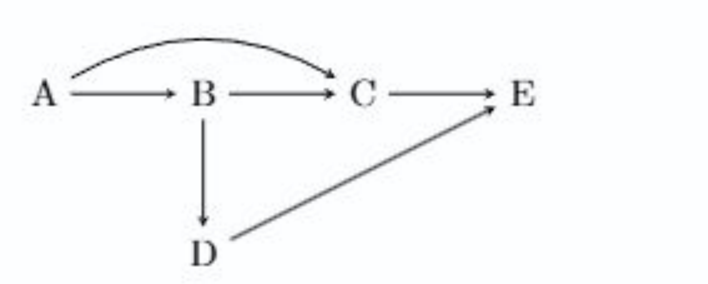
B	0	0	1	1	0
C	0	0	0	0	1
D	0	0	0	0	1
E	0	0	0	0	0

k = A

i = A
j = C

$R^{(0)}$	A	B	C	D	E
A	0	1	1		
B					
C					
D					
E					



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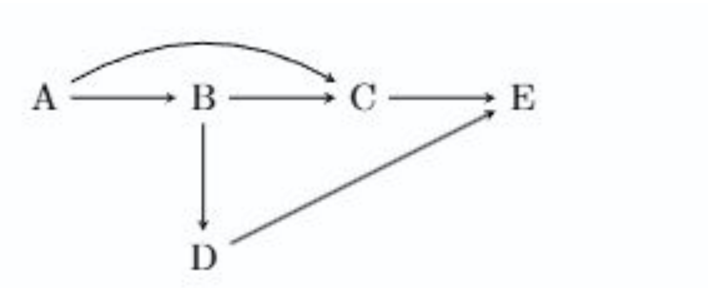
B	0	0	1	1	0
C	0	0	0	0	1
D	0	0	0	0	1
E	0	0	0	0	0

k = A

i = A
j = D

$R^{(0)}$	A	B	C	D	E
A	0	1	1	0	
B					
C					
D					
E					

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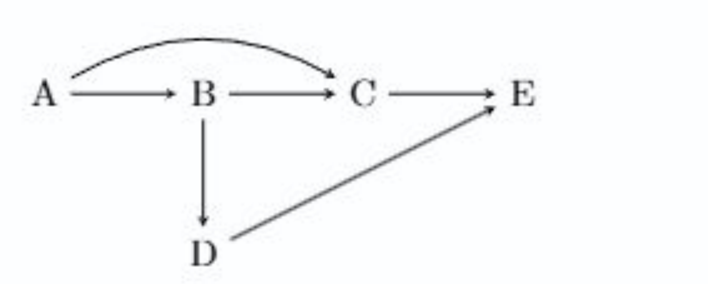
C	0	0	0	0	1
D	0	0	0	0	1
E	0	0	0	0	0

k = A

i = A
j = E

$R^{(0)}$	A	B	C	D	E
A	0	1	1	0	0
B					
C					
D					
E					

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end algorithm
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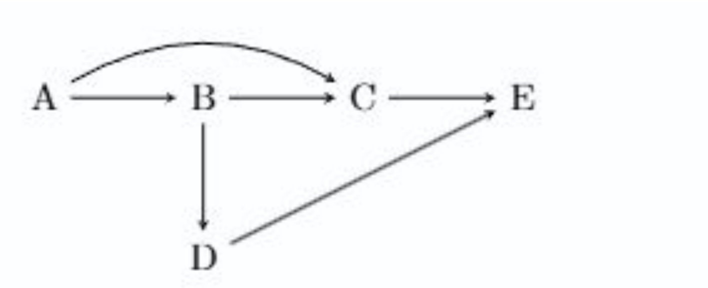
$R^{(-1)}$	A	B	C	D	E
A	0	1	1	0	0
B	0	0	1	1	0
C	0	0	0	0	1
D	0	0	0	0	1
E	0	0	0	0	0

What's the final $R^{(0)}$?

k = A

i = B
j = A

$R^{(0)}$	A	B	C	D	E
A	0	1	1	0	0
B	0				
C					
D					
E					

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      end for
   end for
end for

return R^(n−1)
end algorithm
```

$R^{(-1)}$	A	B	C	D	E
A	0	1	1	0	0
B	0	0	1	1	0
C	0	0	0	0	1
D	0	0	0	0	1
E	0	0	0	0	0

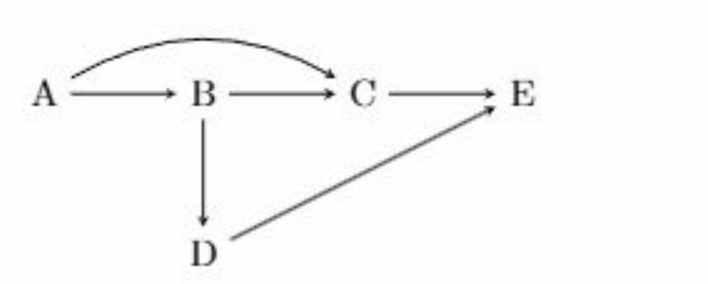
Whats the final $R^{(0)}$? Same as $R^{(-1)}$, why?

k = A

→

i = B
j = A

$R^{(0)}$	A	B	C	D	E
A	0	1	1	0	0
B	0	0	1	1	0
C	0	0	0	0	1
D	0	0	0	0	1
E	0	0	0	0	0

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          end for
       end for
    end for

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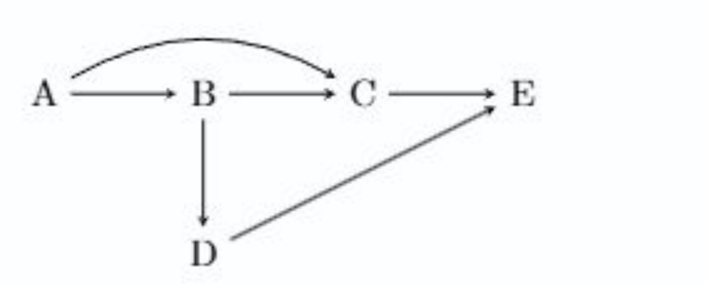
end algorithm
```

$R^{(0)}$	A	B	C	D	E
A	0	1	1	0	0
B	0	0	1	1	0
C	0	0	0	0	1
D	0	0	0	0	1
E	0	0	0	0	0

k = B

i = A
j = A

$R^{(1)}$	A	B	C	D	E
A	0	1	1		
B			1	1	
C					1
D					1
E					

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end algorithm
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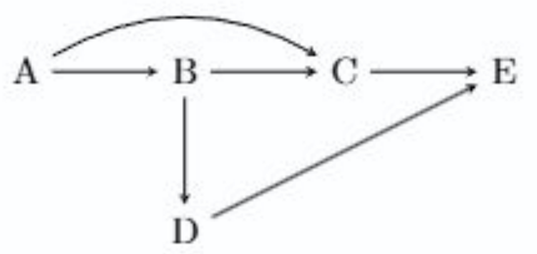
$R^{(0)}$	A	B	C	D	E
A	0	1	1	0	0
B	0	0	1	1	0
C	0	0	0	0	1
D	0	0	0	0	1
E	0	0	0	0	0

k = B

i = A
j = B

$R^{(1)}$	A	B	C	D	E
A	0	1	1		
B			1	1	
C					1
D					1
E					

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          end for
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end algorithm
```

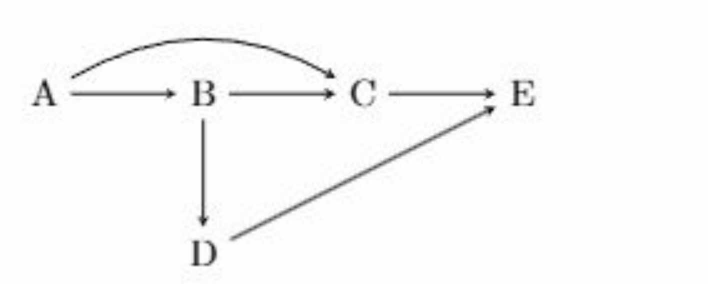
$R^{(0)}$	A	B	C	D	E
A	0	1	1	0	0
B	0	0	1	1	0
C	0	0	0	0	1
D	0	0	0	0	1
E	0	0	0	0	0

k = B

→

i = A
j = C

$R^{(1)}$	A	B	C	D	E
A	0	1	1		
B			1	1	
C					1
D					1
E					

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          end for
       end for
    end for

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end algorithm
```

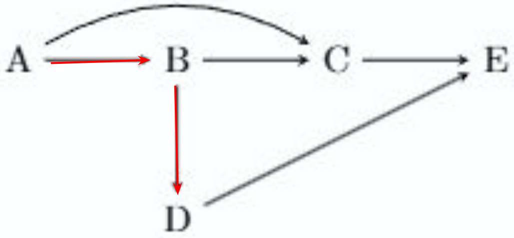
$R^{(0)}$	A	B	C	D	E
A	0	1	1	0	0
B	0	0	1	1	0
C	0	0	0	0	1
D	0	0	0	0	1
E	0	0	0	0	0

k = B

→

i = A
j = D

$R^{(1)}$	A	B	C	D	E
A	0	1	1		
B			1	1	
C					1
D					1
E					

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    end for
end for

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end algorithm
```

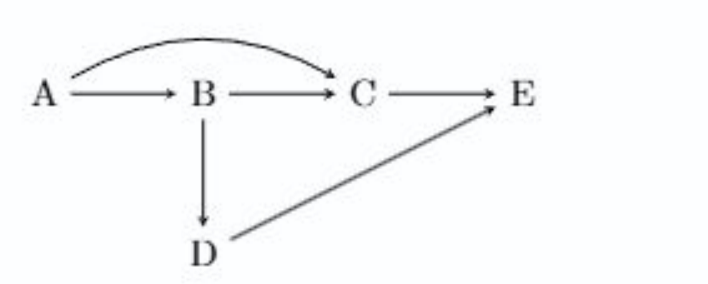
$R^{(0)}$	A	B	C	D	E
A	0	1	1	0	0
B	0	0	1	1	0
C	0	0	0	0	1
D	0	0	0	0	1
E	0	0	0	0	0

k = B

→

i = A
j = D

$R^{(1)}$	A	B	C	D	E
A	0	1	1	1	
B			1	1	
C					1
D					1
E					



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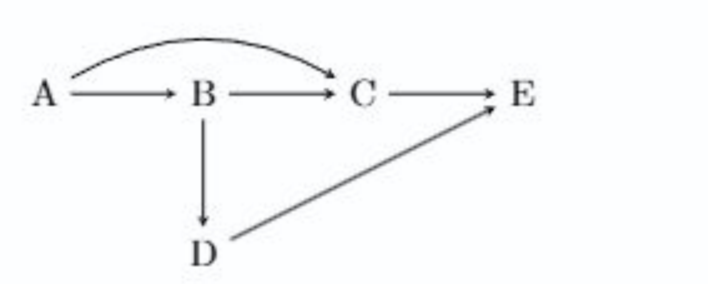
end algorithm

$R^{(0)}$	A	B	C	D	E
A	0	1	1	0	0
B	0	0	1	1	0
C	0	0	0	0	1
D	0	0	0	0	1
E	0	0	0	0	0

k = B

i = A
j = E

$R^{(1)}$	A	B	C	D	E
A	0	1	1	1	?
B			1	1	
C					1
D					1
E					




```
    R^(−1) ← M
    n ← |V|

    for k from 0 to n−1 do
        for i from 0 to n−1 do
            for j from 0 to n−1 do
                R^(k)[i,j] ← R^(k−1)[i,j] or (R^(k−1)[i,k] and R^(k−1)[k,j])
            end for
        end for
    end for

    return R^(n−1)
end algorithm
```

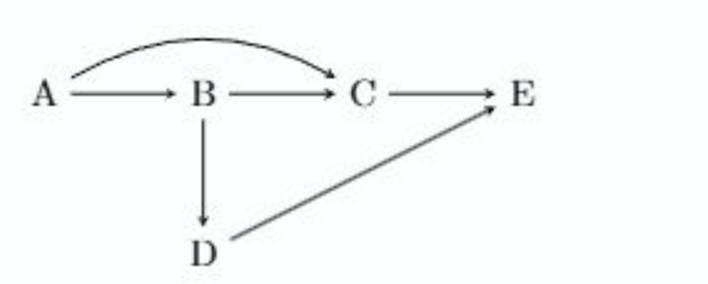
$R^{(0)}$	A	B	C	D	E
A	0	1	1	0	0
B	0	0	1	1	0
C	0	0	0	0	1
D	0	0	0	0	1
E	0	0	0	0	0

k = B

→

i = A
j = E

$R^{(1)}$	A	B	C	D	E
A	0	1	1	1	0
B			1	1	
C					1
D					1
E					




```
R^(-1) ← M
n ← |V|

for k from 0 to n−1 do
   for i from 0 to n−1 do
      for j from 0 to n−1 do
         R^(k)[i,j] ← R^(k−1)[i,j] or (R^(k−1)[i,k] and R^(k−1)[k,j])
      end for
   end for
end for

return R^(n−1)
end algorithm
```

$R^{(0)}$	A	B	C	D	E
A	0	1	1	0	0
B	0	0	1	1	0
C	0	0	0	0	1
D	0	0	0	0	1
E	0	0	0	0	0

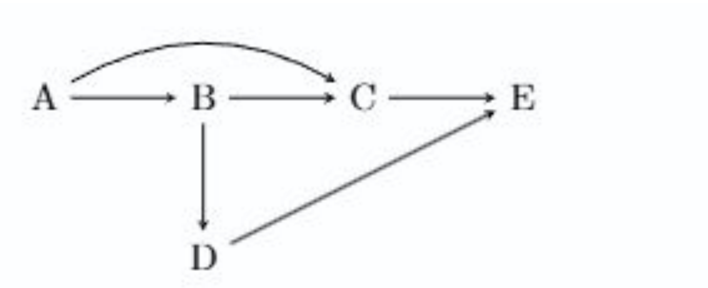
What's the final $R^{(1)}$?

k = B

→

i = B
j = A

$R^{(1)}$	A	B	C	D	E
A	0	1	1	1	0
B			1	1	
C					1
D					1
E					




```
R^(−1) ← M
n ← |V|

for k from 0 to n−1 do
   for i from 0 to n−1 do
      for j from 0 to n−1 do
         R^(k)[i,j] ← R^(k−1)[i,j] or (R^(k−1)[i,k] and R^(k−1)[k,j])
      end for
   end for
end for

return R^(n−1)
end algorithm
```

$R^{(0)}$	A	B	C	D	E
A	0	1	1	0	0
B	0	0	1	1	0
C	0	0	0	0	1
D	0	0	0	0	1
E	0	0	0	0	0

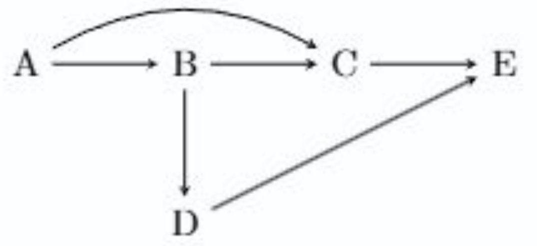
What's the final $R^{(1)}$?

k = B

→

i = B
j = A

$R^{(1)}$	A	B	C	D	E
A	0	1	1	1	0
B	0	0	1	1	1
C	0	0	0	0	1
D	0	0	0	0	1
E	0	0	0	0	0



$R^{(-1)} \leftarrow M$
$n \leftarrow |V|$

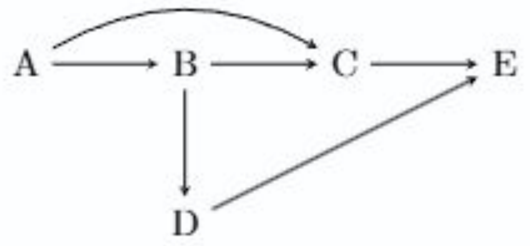
```
for k from 0 to n-1 do
   for i from 0 to n-1 do
      for j from 0 to n-1 do
         R^(k)[i,j] ← R^(k-1)[i,j] or (R^(k-1)[i,k] and R^(k-1)[k,j])
      end for
   end for
end for

return R^(n-1)
end algorithm
```

$R^{(1)}$	A	B	C	D	E
A	0	1	1	1	0
B	0	0	1	1	1
C	0	0	0	0	1
D	0	0	0	0	1
E	0	0	0	0	0

What's the final $R^{(2)}$?

k = C →

$R^{(2)}$	A	B	C	D	E
A					
B					
C					
D					
E					



$R^{(-1)} \leftarrow M$
$n \leftarrow |V|$

for k from 0 to $n-1$ **do**
  for i from 0 to $n-1$ **do**
    for j from 0 to $n-1$ **do**
      $R^{(k)}[i,j] \leftarrow R^{(k-1)}[i,j]$ or $(R^{(k-1)}[i,k]$ and $R^{(k-1)}[k,j])$
    end for
  end for
end for

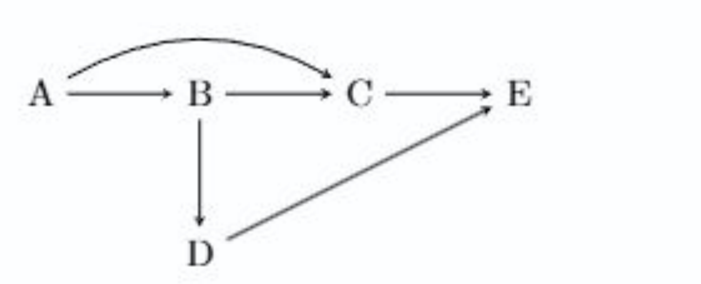
return $R^{(n-1)}$
end algorithm

$R^{(1)}$	A	B	C	D	E
A	0	1	1	1	0
B	0	0	1	1	1
C	0	0	0	0	1
D	0	0	0	0	1
E	0	0	0	0	0

What's the final $R^{(2)}$?

k = C →

$R^{(2)}$	A	B	C	D	E
A	0	1	1	1	1
B	0	0	1	1	1
C	0	0	0	0	1
D	0	0	0	0	1
E	0	0	0	0	0



$$R^{(-1)} \leftarrow M$$
$$n \leftarrow |V|$$

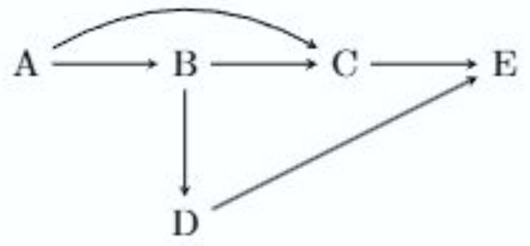
```
for k from 0 to n-1 do
    for i from 0 to n-1 do
        for j from 0 to n-1 do
            R^(k)[i,j] ← R^(k-1)[i,j] or (R^(k-1)[i,k] and R^(k-1)[k,j])
        end for
    end for
end for

return R^(n-1)
end algorithm
```

$R^{(2)}$	A	B	C	D	E
A	0	1	1	1	1
B	0	0	1	1	1
C	0	0	0	0	1
D	0	0	0	0	1
E	0	0	0	0	0

What's the final $R^{(3)}$?

k = D →

$R^{(3)}$	A	B	C	D	E
A					
B					
C					
D					
E					



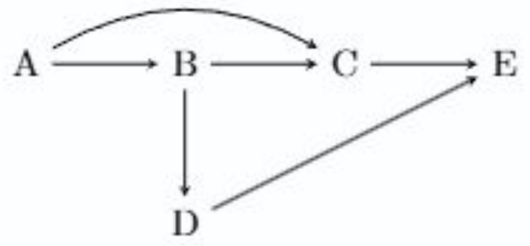

```
R^(-1) ← M
n ← |V|

for k from 0 to n−1 do
   for i from 0 to n−1 do
      for j from 0 to n−1 do
         R^(k)[i,j] ← R^(k−1)[i,j] or (R^(k−1)[i,k] and R^(k−1)[k,j])
      end for
   end for
end for

return R^(n−1)
end algorithm
```

$R^{(2)}$	A	B	C	D	E
A	0	1	1	1	1
B	0	0	1	1	1
C	0	0	0	0	1
D	0	0	0	0	1
E	0	0	0	0	0

What's the final $R^{(3)}$?

k = D →

$R^{(3)}$	A	B	C	D	E
A	0	1	1	1	1
B	0	0	1	1	1
C	0	0	0	0	1
D	0	0	0	0	1
E	0	0	0	0	0



$$R^{(-1)} \leftarrow M$$
$$n \leftarrow |V|$$

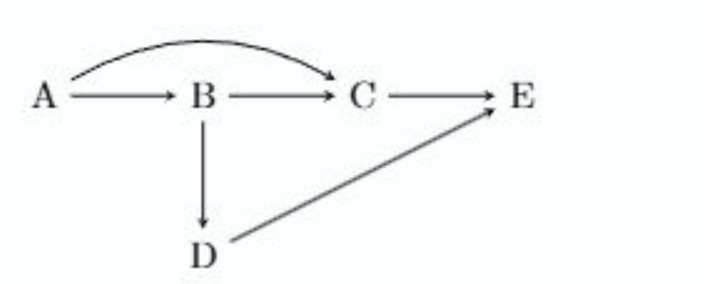
```
for k from 0 to n-1 do
   for i from 0 to n-1 do
      for j from 0 to n-1 do
         R^(k)[i,j] ← R^(k-1)[i,j] or (R^(k-1)[i,k] and R^(k-1)[k,j])
      end for
   end for
end for

return R^(n-1)
end algorithm
```

$R^{(3)}$	A	B	C	D	E
A	0	1	1	1	1
B	0	0	1	1	1
C	0	0	0	0	1
D	0	0	0	0	1
E	0	0	0	0	0

What's the final $R^{(4)}$?

k = E →

$R^{(4)}$	A	B	C	D	E
A	0	1	1	1	1
B	0	0	1	1	1
C	0	0	0	0	1
D	0	0	0	0	1
E	0	0	0	0	0

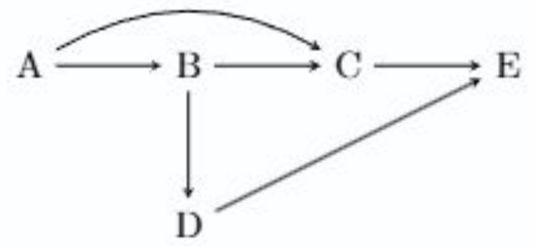


$R^{(4)}$	A	B	C	D	E
A	0	1	1	1	1
B	0	0	1	1	1
C	0	0	0	0	1
D	0	0	0	0	1
E	0	0	0	0	0

Summary

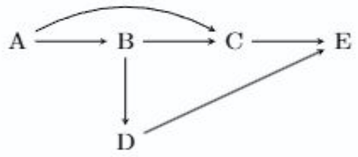
For each k = A,B,C,D,E:
    For each i = …:
        For each j = …:
            Check if there is a path between i and j through k

$R^{(-1)}$: Adj Matrix
$R^{(0/A)}$: $R^{(-1)}$ + (paths through A)
$R^{(1/B)}$: $R^{(0/A)}$+ (paths through B)
$R^{(2/C)}$: $R^{(1/B)}$ + (paths through C)
$R^{(3/D)}$: $R^{(2/C)}$+ (paths through D)
$R^{(4/E)}$: $R^{(3/D)}$+ (paths through E)



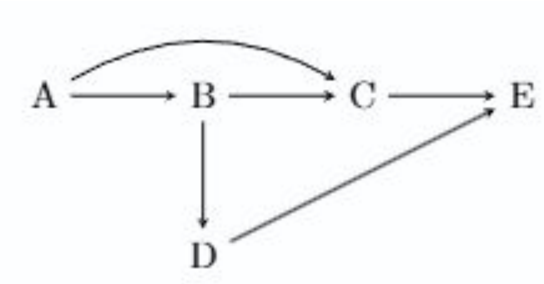
Question 2

Consider the directed graph $G = (V, E)$ given below:



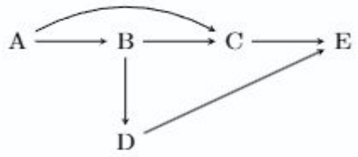
3. Draw the graph representation of the transitive closure of G.

$R^{(4)}$	A	B	C	D	E
A	0	1	1	1	1
B	0	0	1	1	1
C	0	0	0	0	1
D	0	0	0	0	1
E	0	0	0	0	0



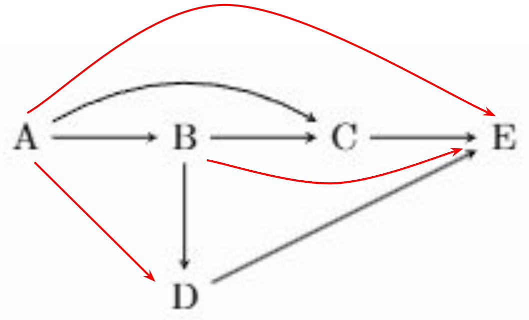
Question 2

Consider the directed graph $G = (V, E)$ given below:

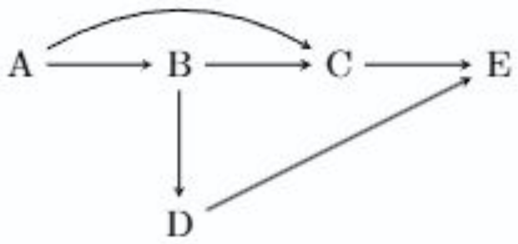


4. Determine the reachability of each node in G.

$R^{(4)}$	A	B	C	D	E
A	0	1	1	1	1
B	0	0	1	1	1
C	0	0	0	0	1
D	0	0	0	0	1
E	0	0	0	0	0



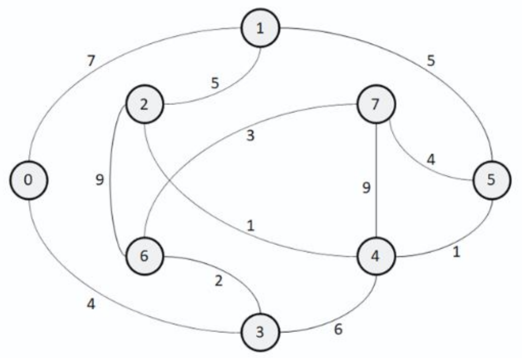
The transitive closure can help here



5. Identify if G is strongly connected. If not, can you add one edge to make G become a strongly connected graph?

Question 3

Consider the following graph G:



v	deg(v)
1	
2	
3	
4	
5	
6	
7	

Let G_d be a directed graph using the vertices of G. For a pair of vertices u and v connected by an edge in G, their respective directed edge in G_d is as follows:

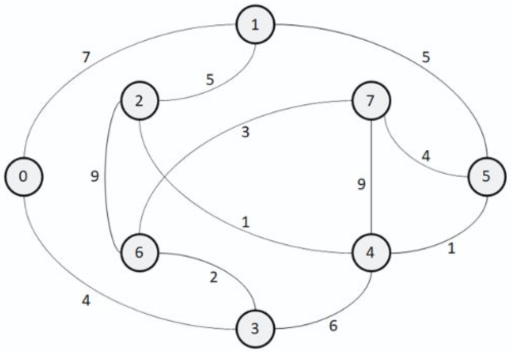
$$\text{Edge with vertices } u \text{ and } v = \begin{cases} (u,v), & \deg(u) < \deg(v) \vee (\deg(u) = \deg(v) \wedge u < v) \\ (v,u), & \text{Otherwise} \end{cases}$$

Let's draw G_d

First, calculate deg(v)

Question 3

Consider the following graph G:

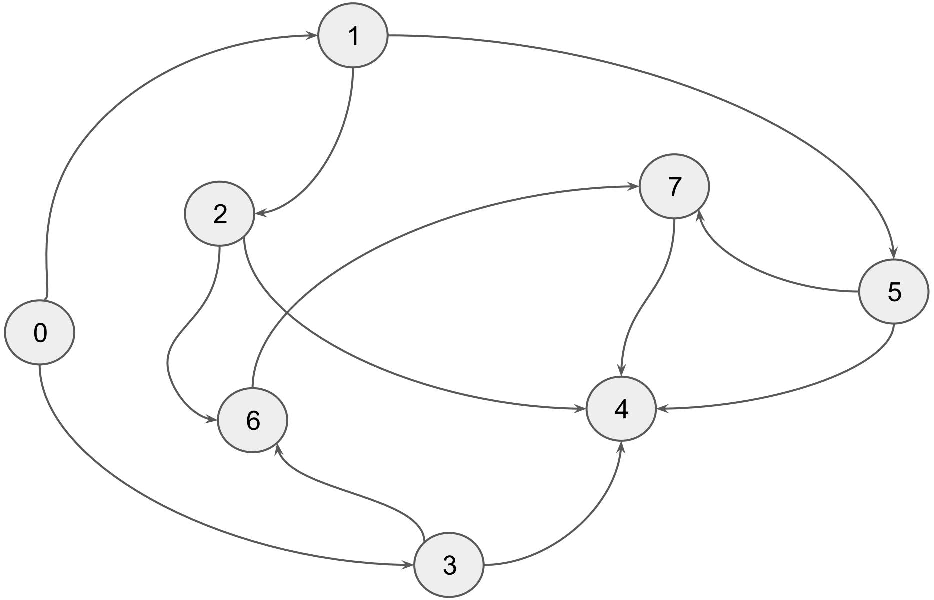


Let G_d be a directed graph using the vertices of G. For a pair of vertices u and v connected by an edge in G, their respective directed edge in G_d is as follows:

$$\text{Edge with vertices } u \text{ and } v = \begin{cases} (u,v), & \deg(u) < \deg(v) \vee (\deg(u) = \deg(v) \wedge u < v) \\ (v,u), & \text{Otherwise} \end{cases}$$

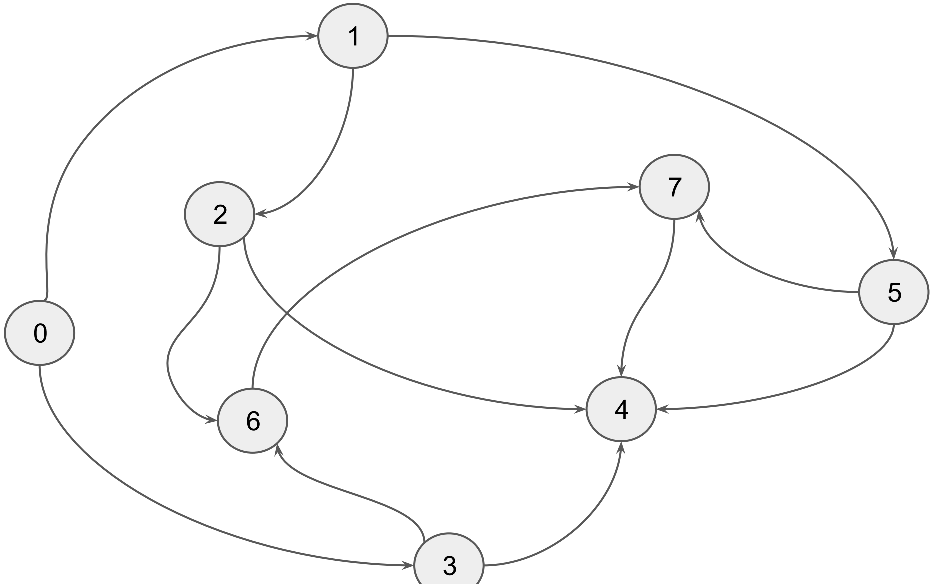
Let's draw G_d

v	deg(v)
1	3
2	3
3	3
4	4
5	3
6	3
7	3



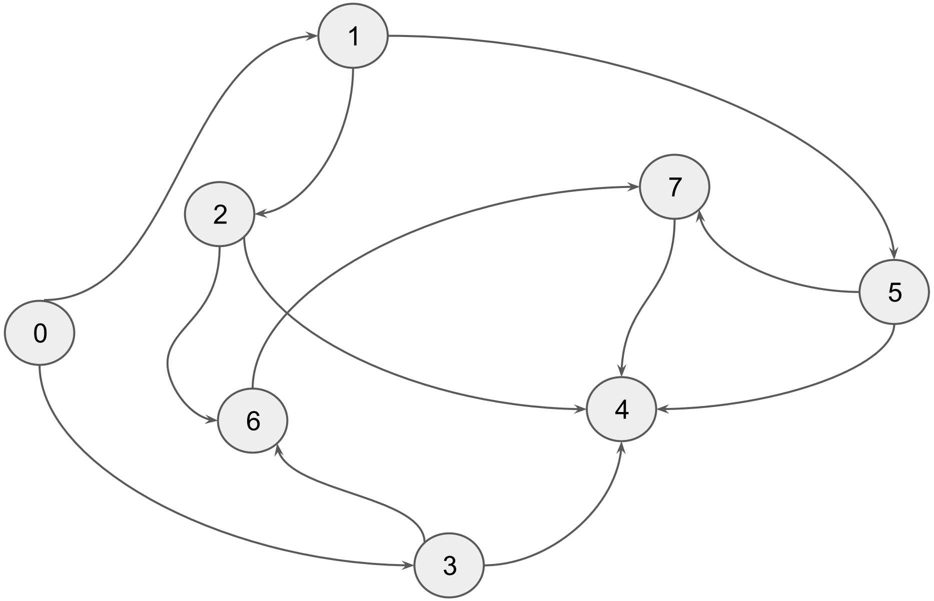
1. Is G_d strongly connected? If yes, explain why. Otherwise, list the minimum number of edges required to make G_d strongly connected.

v	deg(v)
1	3
2	3
3	3
4	4
5	3
6	3
7	3



1. Is G_d strongly connected? If yes, explain why. Otherwise, list the minimum number of edges required to make G_d strongly connected.

If we run Warshall..

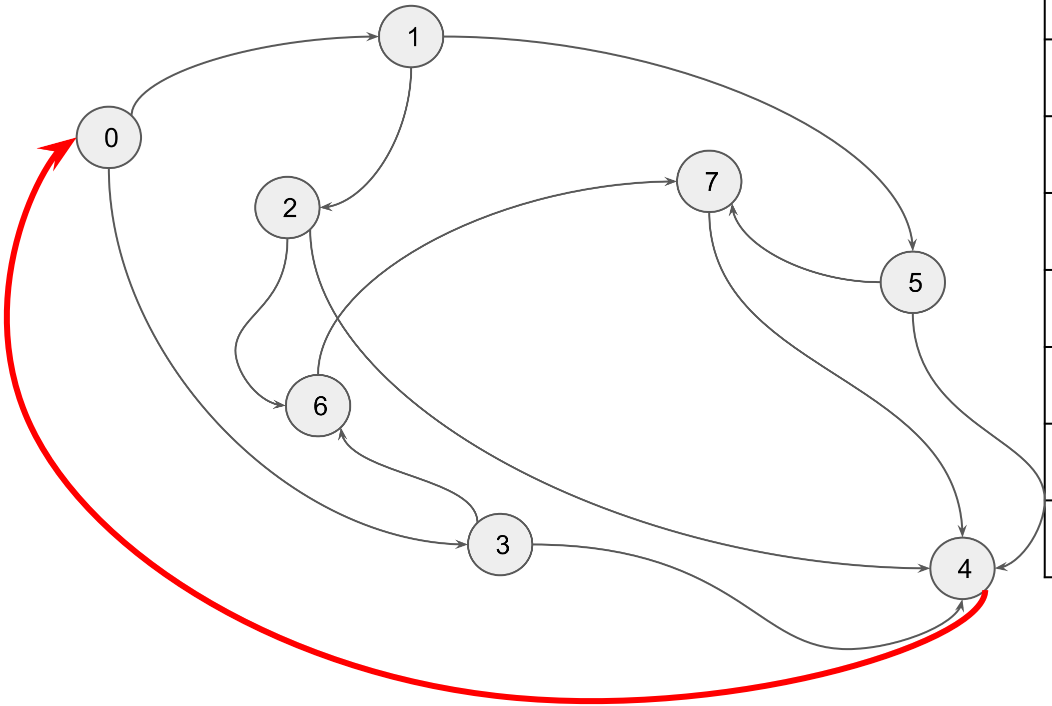


u/v	0	1	2	3	4	5	6	7
0		1	1	1	1	1	1	1
1			1		1	1	1	1
2					1		1	1
3					1		1	1
4								
5					1			1
6					1			1
7					1			

Observations:

1. Is G_d strongly connected? If yes, explain why. Otherwise, list the minimum number of edges required to make G_d strongly connected.

If we run Warshall..

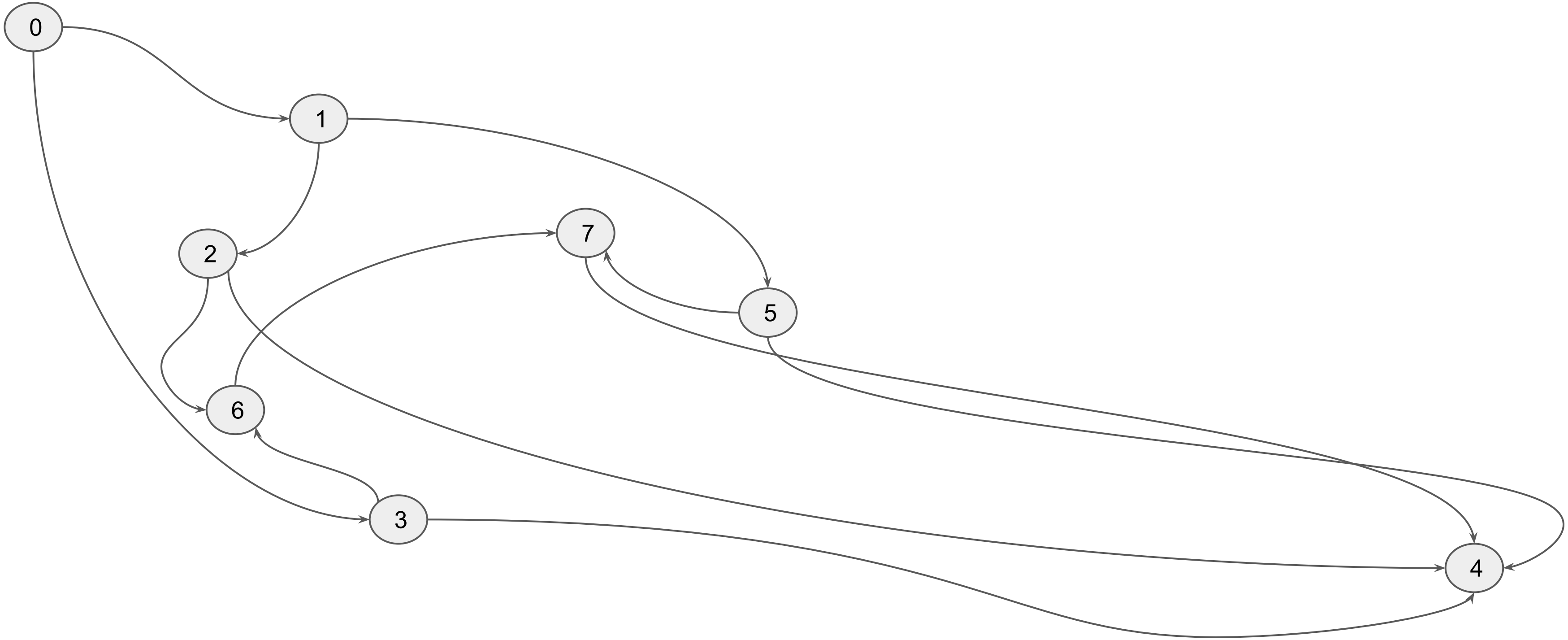


u/v	0	1	2	3	4	5	6	7
0		1	1	1	1	1	1	1
1			1		1	1	1	1
2					1		1	1
3					1		1	1
4								
5					1			1
6					1			1
7					1			

Adding (4,0) makes strongly connected

2. Show all the topological orderings of G_d.

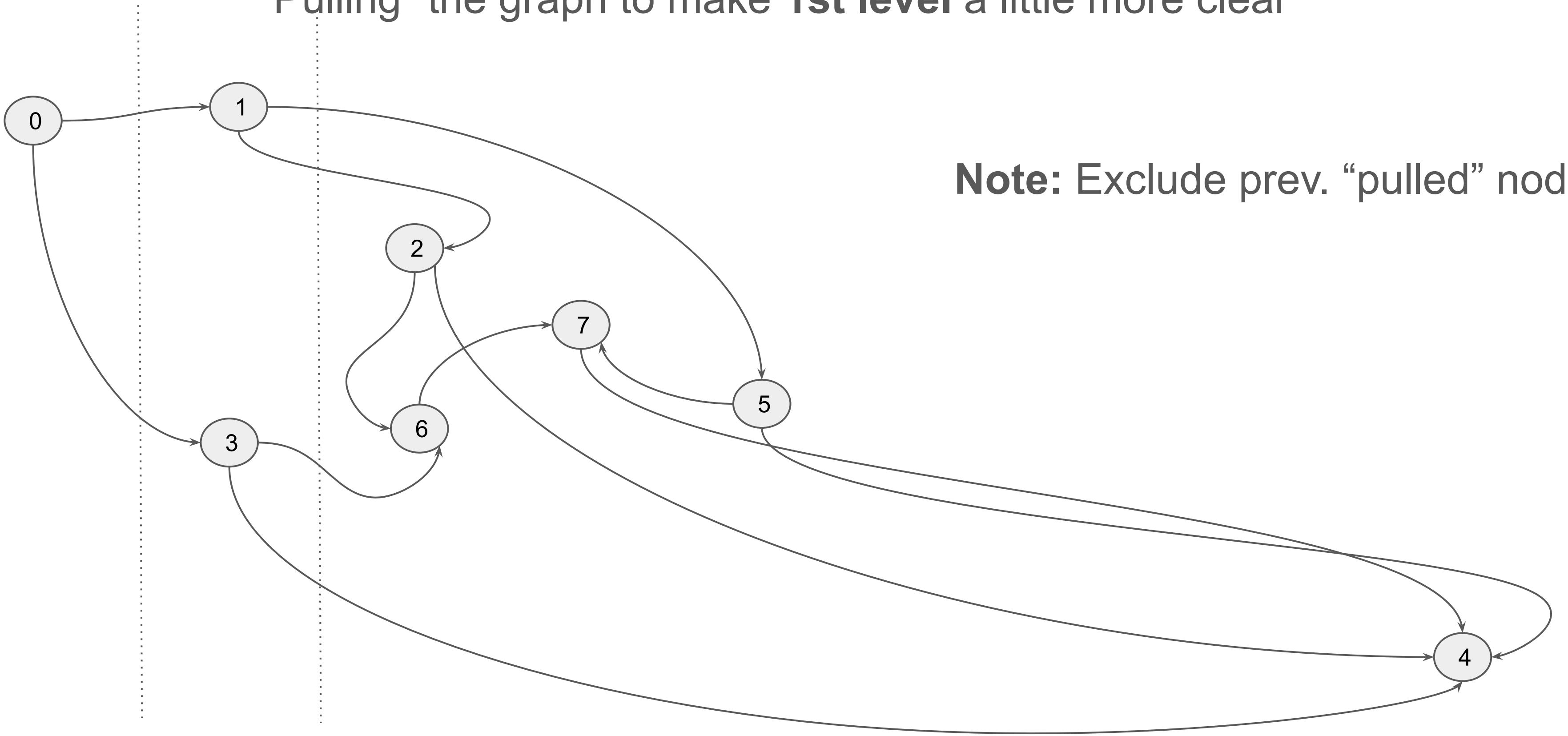
“Pulling” the graph to make source/sink a little more clear



2. Show all the topological orderings of G_d.

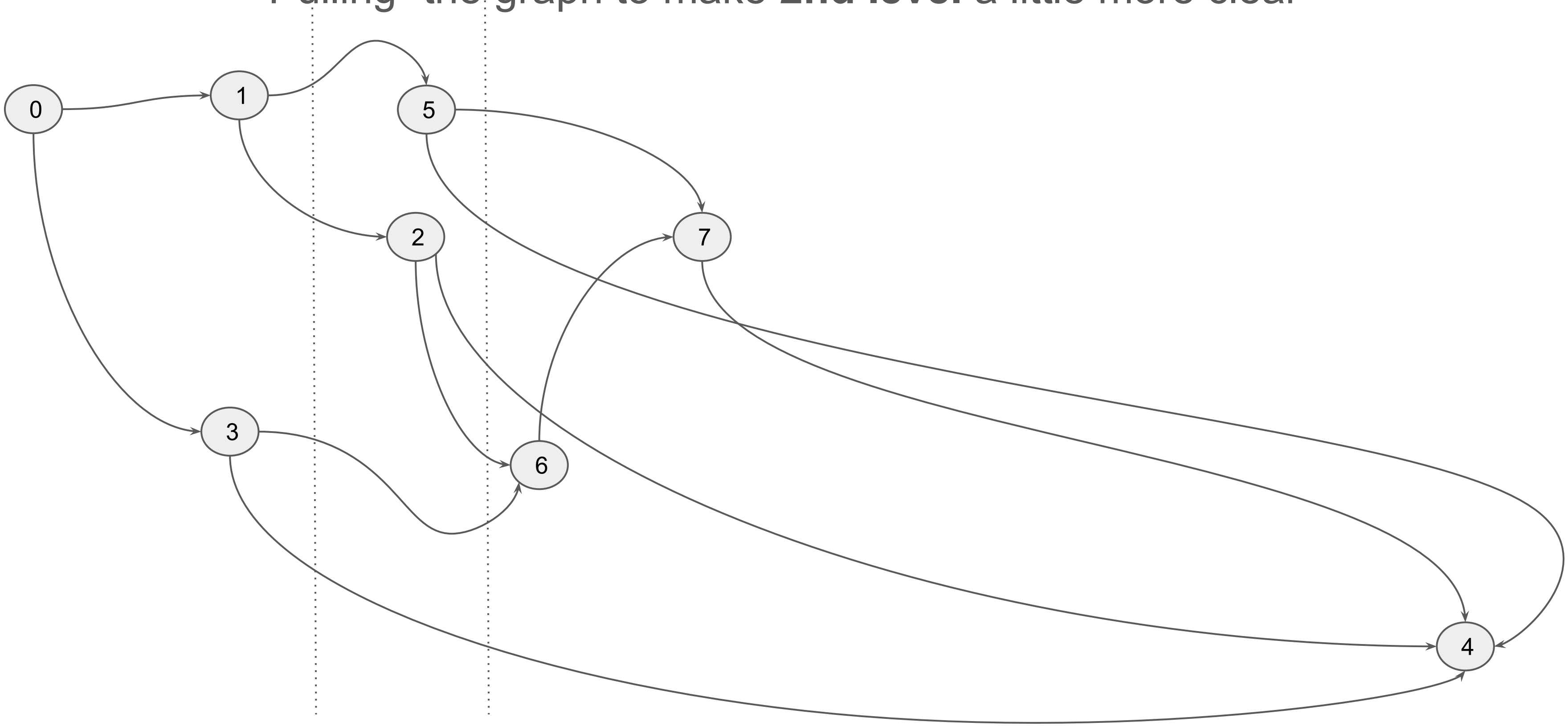
"Pulling" the graph to make **1st level** a little more clear

Note: Exclude prev. "pulled" nodes



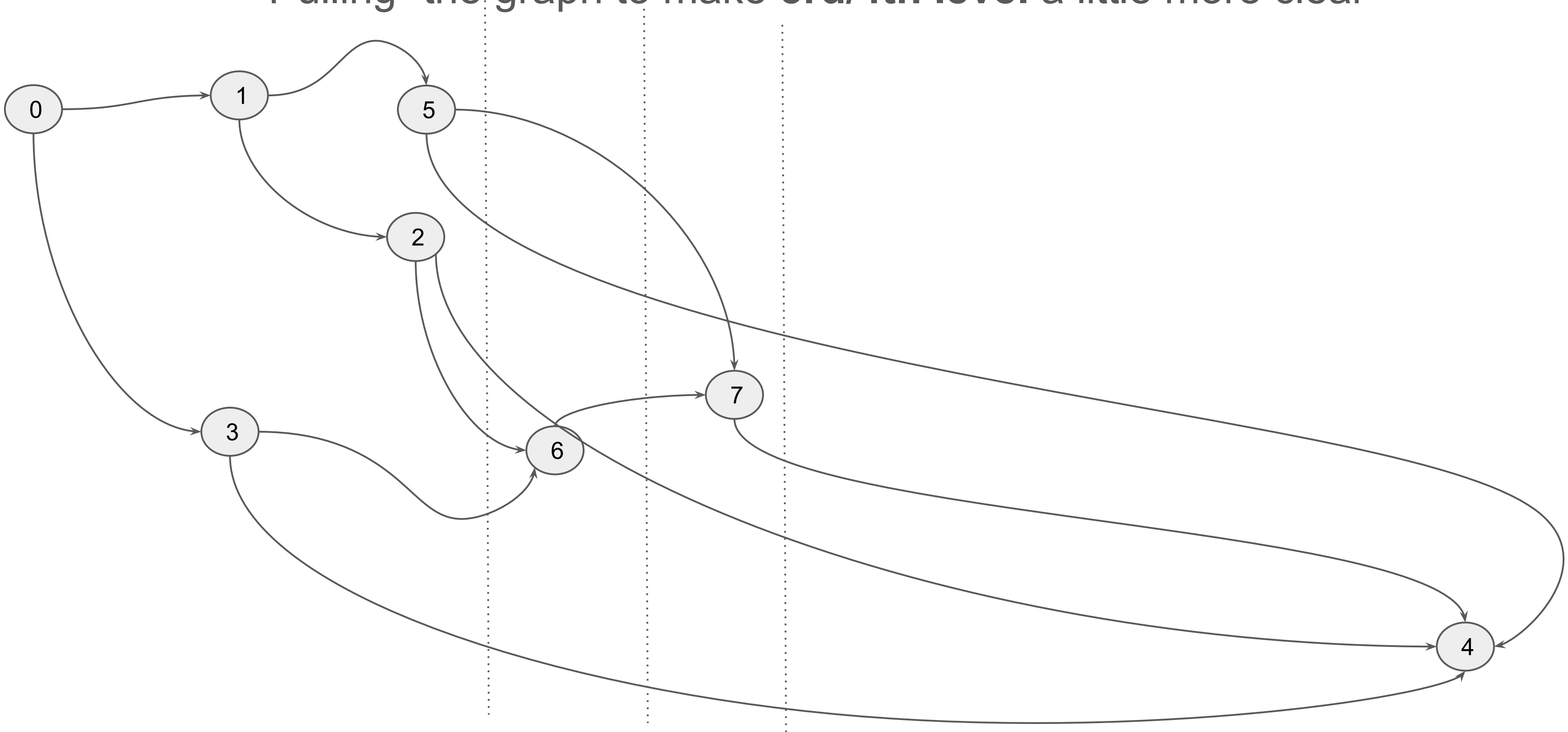
2. Show all the topological orderings of G_d.

"Pulling" the graph to make **2nd level** a little more clear



2. Show all the topological orderings of G_d.

"Pulling" the graph to make **3rd/4th level** a little more clear



2. Show all the topological orderings of G_d.

I can write out the topo orderings now

Any node on the same level can go in either order