PSO 3

Exam this **Friday**. Topics are

1. Run time Expressions/Asymptotic Analysis
2. Array
3. Linked List
4. Stack n’ Queue
5. Trees
6. Heaps & Heap Sort

Question 1

(Linked List) Consider a sorted circular doubly linked list of N numbers where the head element points to the smallest element in the list. Provide the asymptotic complexity in big-Θ with a brief explanation (including assumptions and analysis for each case, if there is more than one) for the following operations.

1. Inserting an element in its sorted position.

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2. Finding the smallest element in the list.

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3. Finding the 3^{rd} - largest element in the list.

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4. Finding the median in the list.

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1. Inserting an element in its sorted position.
2. Finding the smallest element in the list.
3. Finding the 3^{rd} - largest element in the list.
4. Finding the median in the list.

(Binary Tree)

(1) A full binary tree cannot have which of the following number of nodes?

A. 3

B. 7

C. 11

D. 12

E. 15

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Definition of a full binary tree?

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A. 3
B. 7
C. 11
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examples

Definition of a full binary tree?

Every node is either a

- leaf or,
- inner node with two children

What is the answer?

(2) Given the number of nodes $n = 7$, how many distinct shapes can a full binary tree have?

A. 3

B. 4

C. 5

D. 6

E. 7

How to proceed?

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Every answer is at most 7.. Just draw them all out!

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A. 3

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How to proceed?

Every answer is at most 7.. Just draw them all out!

(3) The number of leaf nodes is always greater than the number of internal nodes in a full binary tree.

A. True

B. False

Thoughts?

(3) The number of leaf nodes is always greater than the number of internal nodes in a full binary tree.

A. True

B. False

If the thought isn't a strong 'yes' then draw examples

(4) The number of leaf nodes is always greater than the number of internal nodes in a complete binary tree.

A. True

B. False

Definition of a *complete* binary tree?

(4) The number of leaf nodes is always greater than the number of internal nodes in a complete binary tree.

A. True

B. False

Definition of a *complete* binary tree?

- Every level of the tree except the last is complete

(5) Given the number of nodes in a full binary tree, the number of its leaf nodes is determined.

A. True

B. False

(Stack and Queue)

Design a stack using two queues satisfying the following requirements

1. Pushing an element to the stack takes no more than $O(1)$ operations.
2. Popping from the stack takes no more than $O(1)$ operations if performed after a push.
3. Popping from the stack takes no more than $O(n)$ operations if performed after another pop, where n is the number of elements in the data structure.

Assume `Queue` interface

- `q = Queue.init()`
- `q.enq(x)`
- `x = q.deq()`
- `q.size()`

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- `q = Queue.init()`
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Implement `Stack` interface

- `s = Stack.init()`
- `s.push(x)`
- `x = s.pop()`

(Stack and Queue)

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```
def Stack.init():
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```
def Stack.init():
    q1 = Queue.init()
    q2 = Queue.init()
```

(Stack and Queue)

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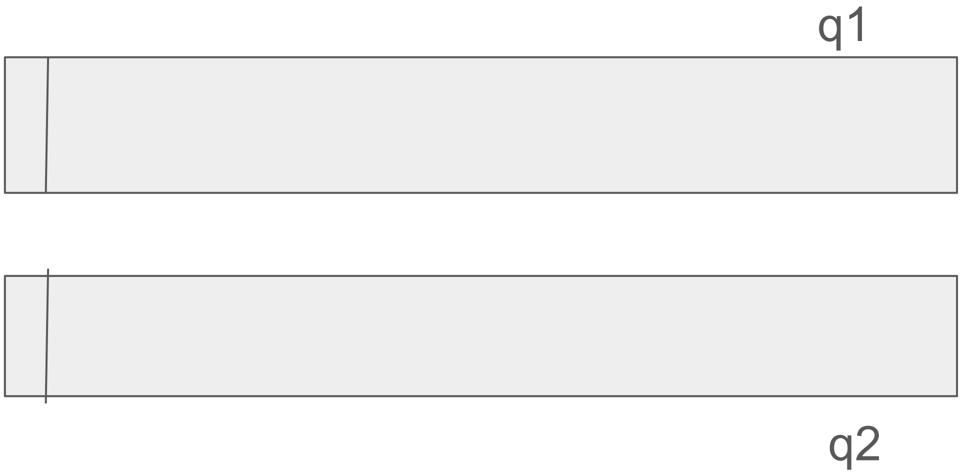
General Strat for these types of problems

- Fulfill conditions incrementally,
- When things break, fix them.
- *Occam's razor*

Example: Starting with the Simplest Push Impl.

1. Pushing an element to the stack takes no more than $O(1)$ operations.

Push(a)
Push(b)
Push(c)
Push(d)



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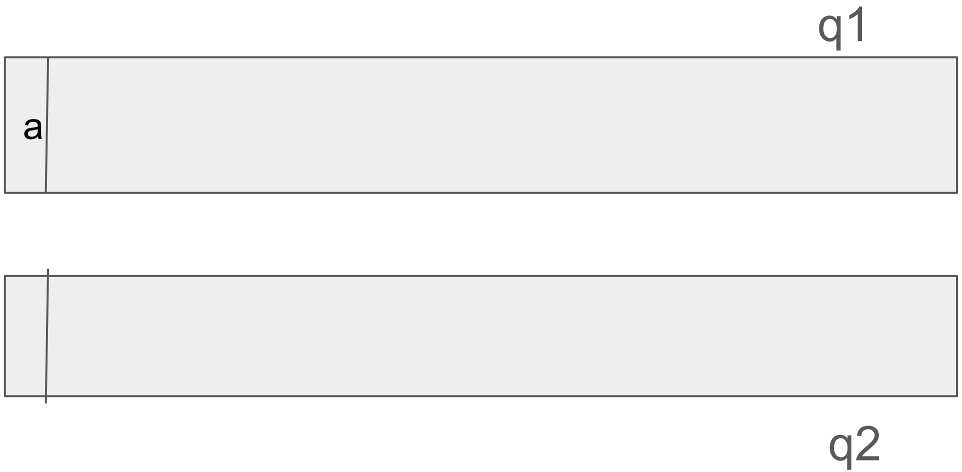

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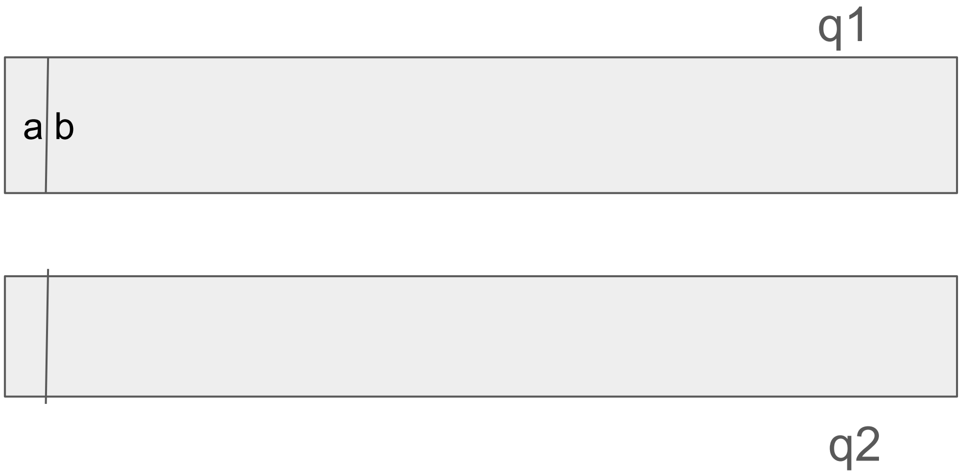

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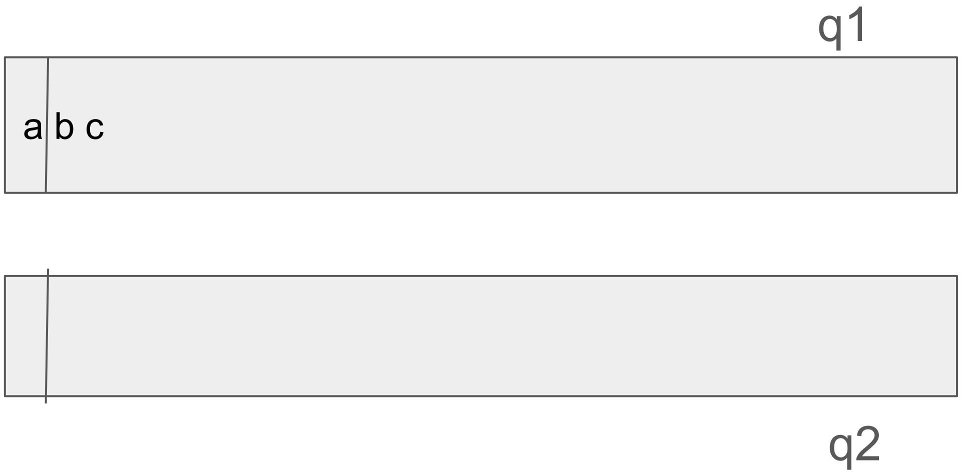

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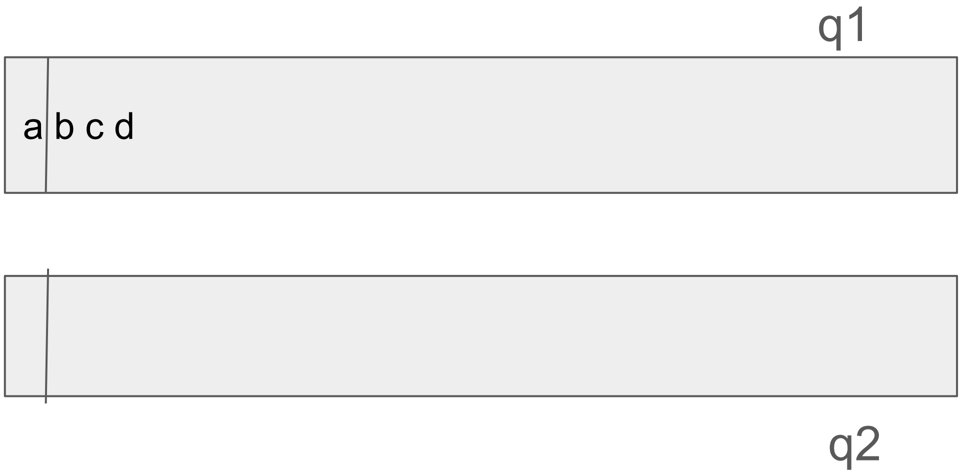
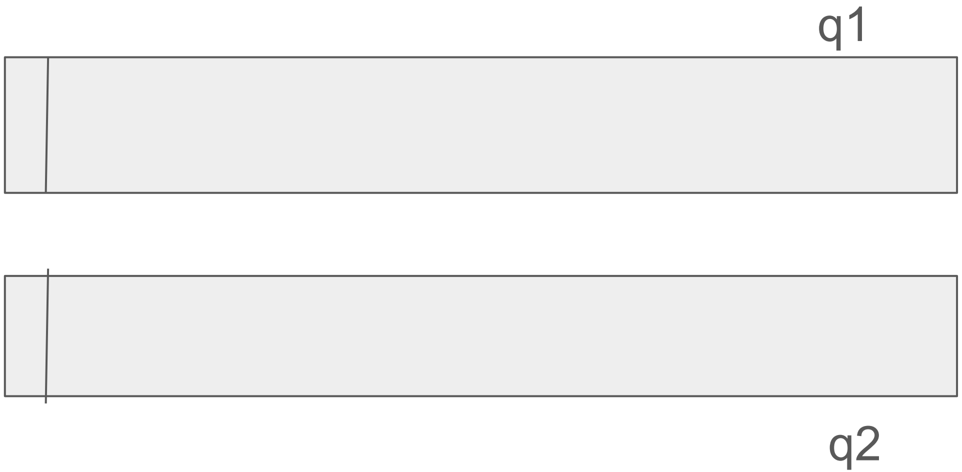


Adding a Pop: Push, Pop?

1. Pushing an element to the stack takes no more than $O(1)$ operations.
2. Popping from the stack takes no more than $O(1)$ operations if performed after a push.

Push(a)
Push(b)
Pop() #should pop b
Push(c)
Pop() # should pop c



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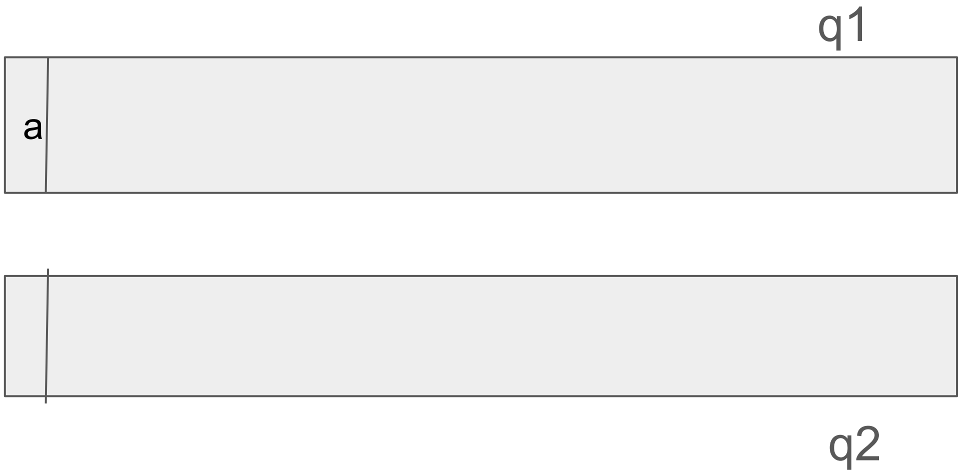

Push, Pop? (use deq?)

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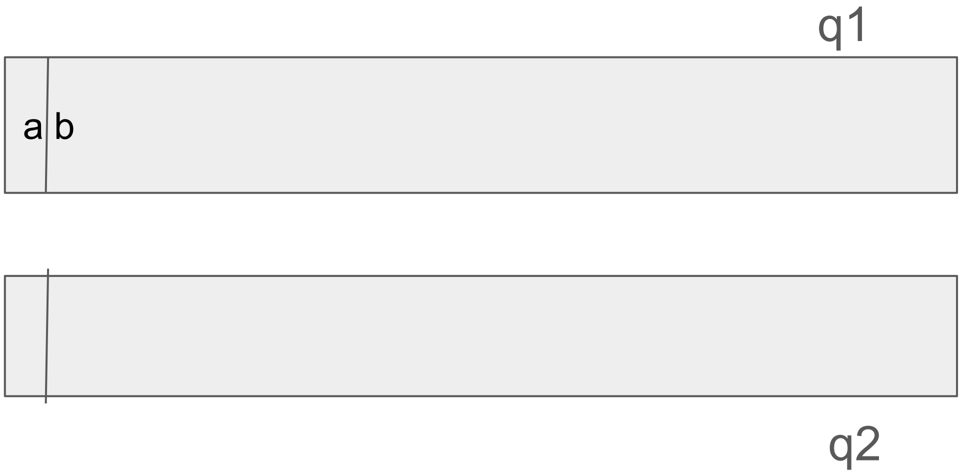

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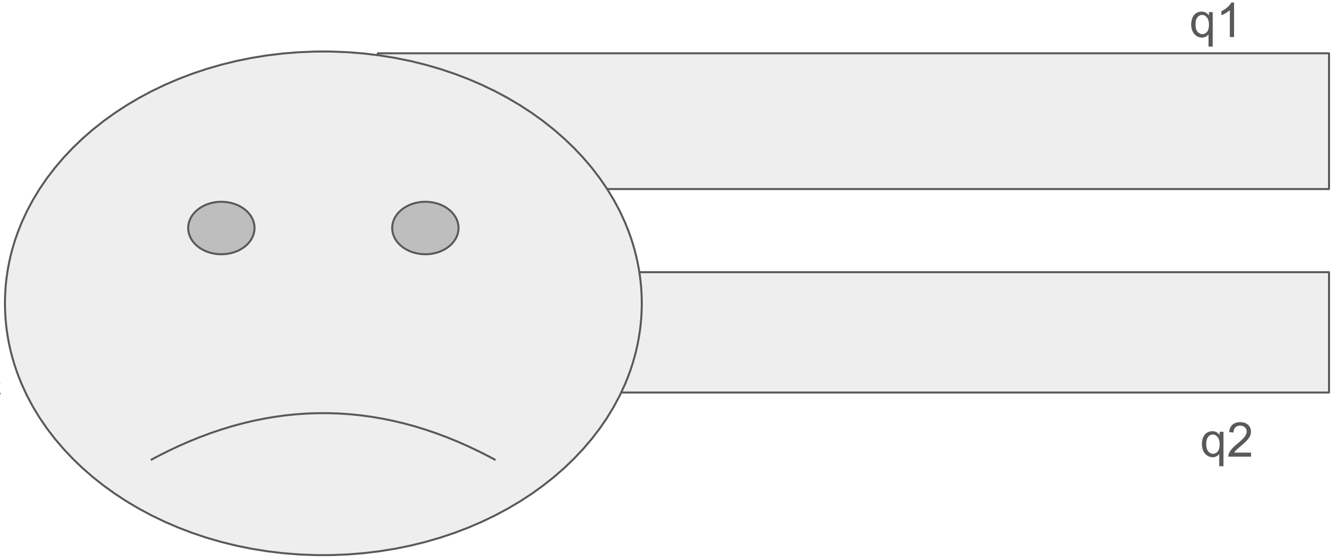

Idea: use q2 to store “last element”

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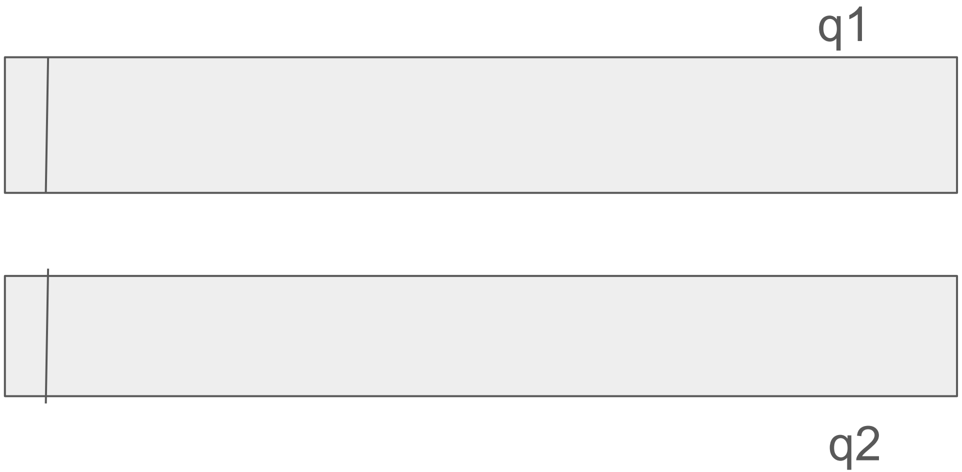

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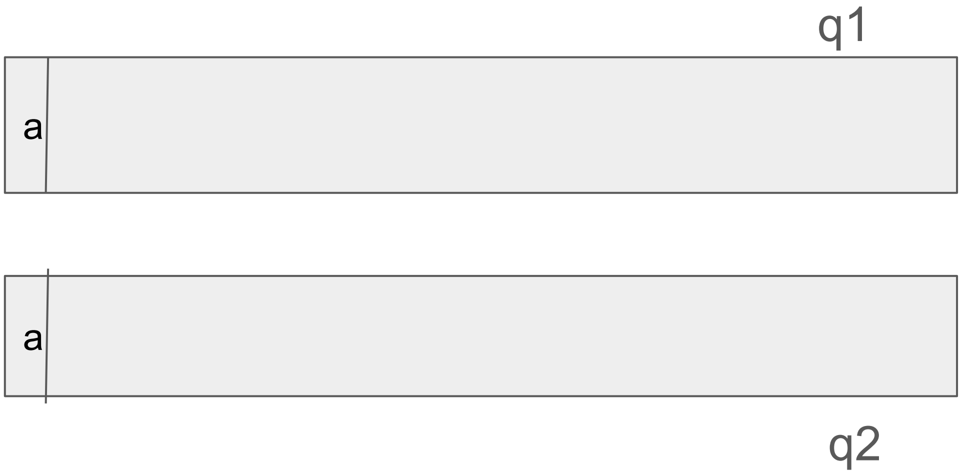

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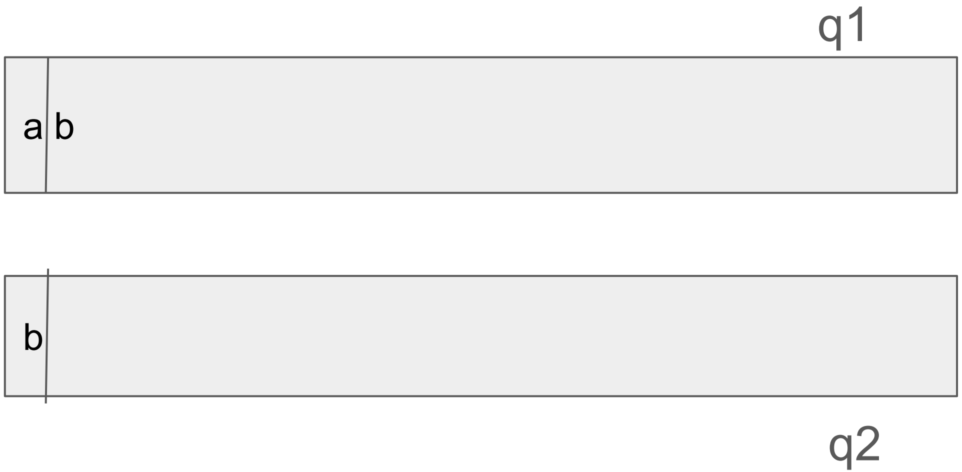

How to implement this?

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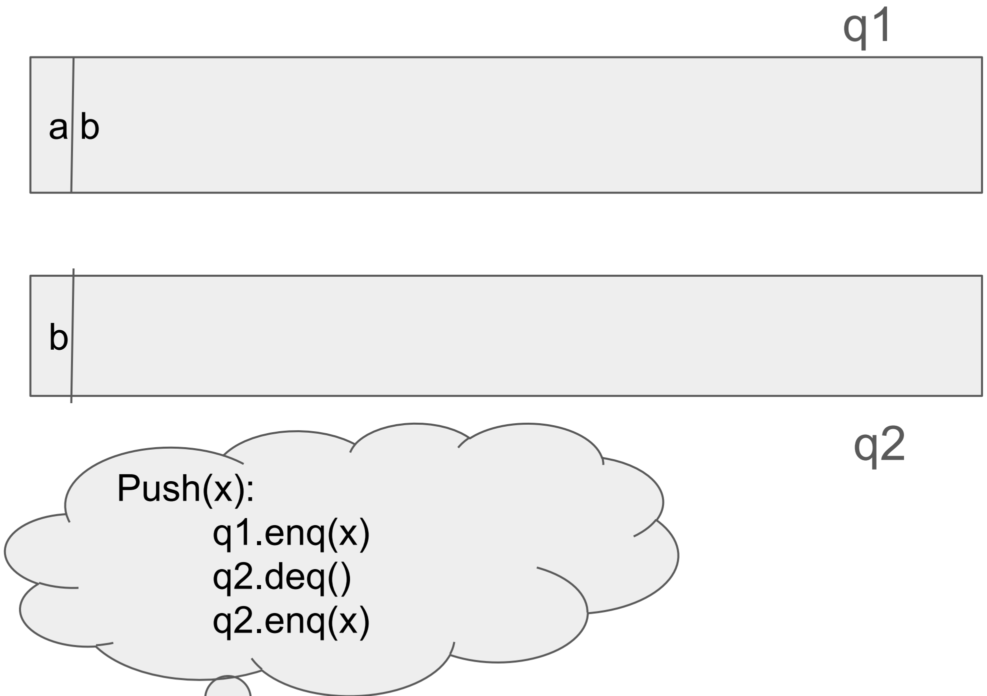

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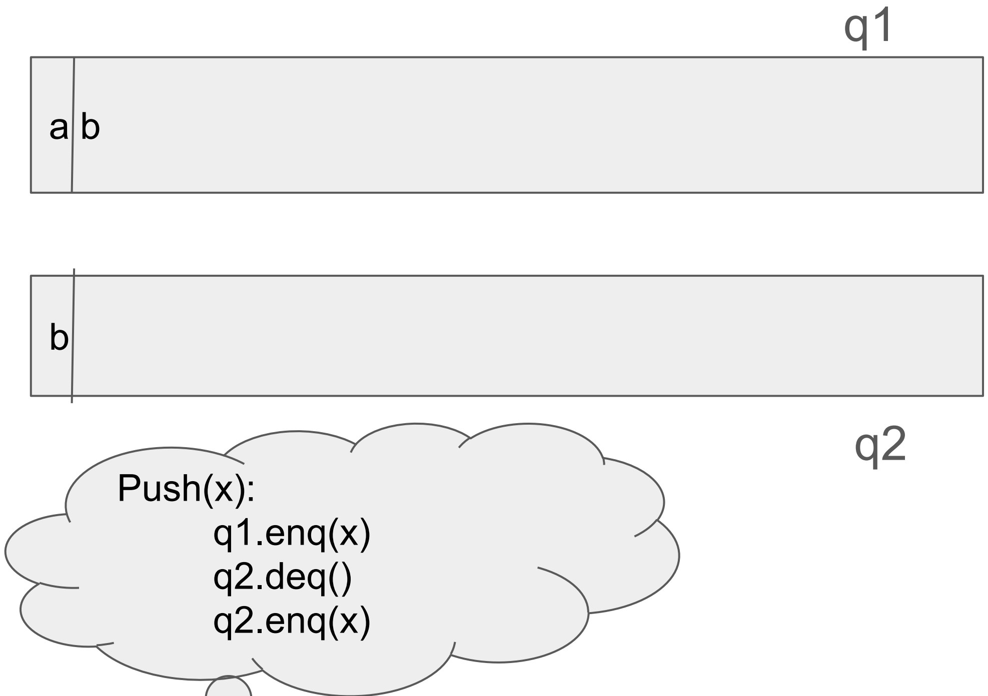

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```
Push(x):
        q1.enq(x)
        q2.deq()
        q2.enq(x)
```

```
Pop():
        If q2.size() > 0:
                Return q2.pop()
```

Pushing after a pop?

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    q1.enq(x)
    q2.deq()
    q2.enq(x)
```

```
Pop():
    If q2.size() > 0:
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```

Pushing after a pop? Only pop if non-empty

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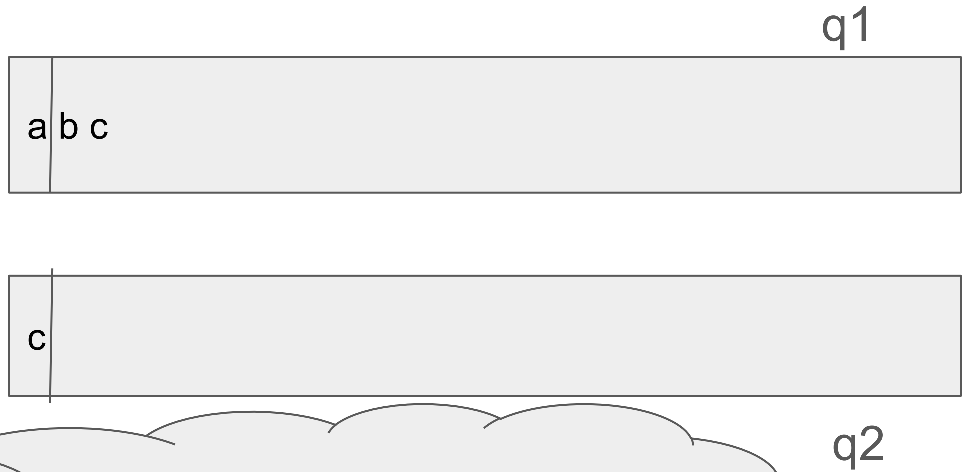
Push(a)
Push(b)
Pop() #should pop b
Push(c)
Pop() # should pop c




```
Push(x):
        q1.enq(x)
        if q2.size > 0: q2.deq()
        q2.enq(x)
```

```
p():
        If q2.size() > 0:
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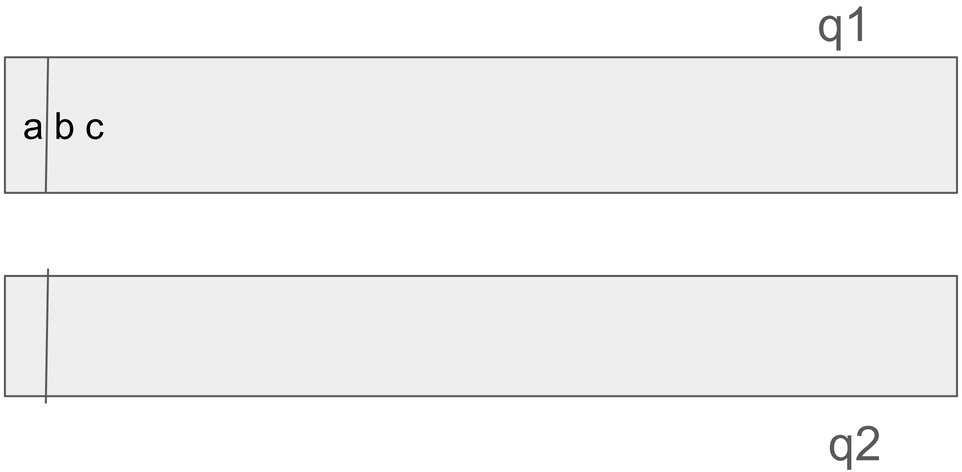

Not exactly a stack, but…
this stack impl is “correct” for the **first two** rules!

Last requirement

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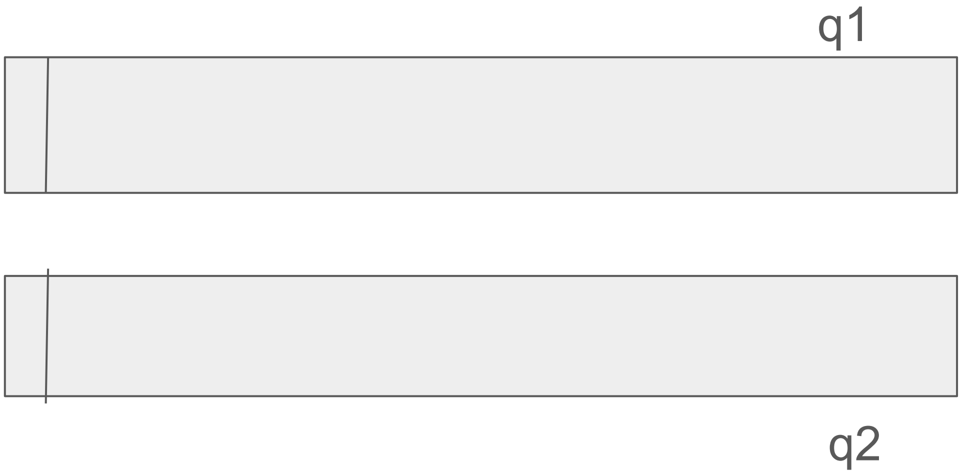

Push(a)
Push(b)
Push(c)
Push(d)
Pop() #should pop d
Pop() # should pop c

Try our implementation as-is

Last requirement

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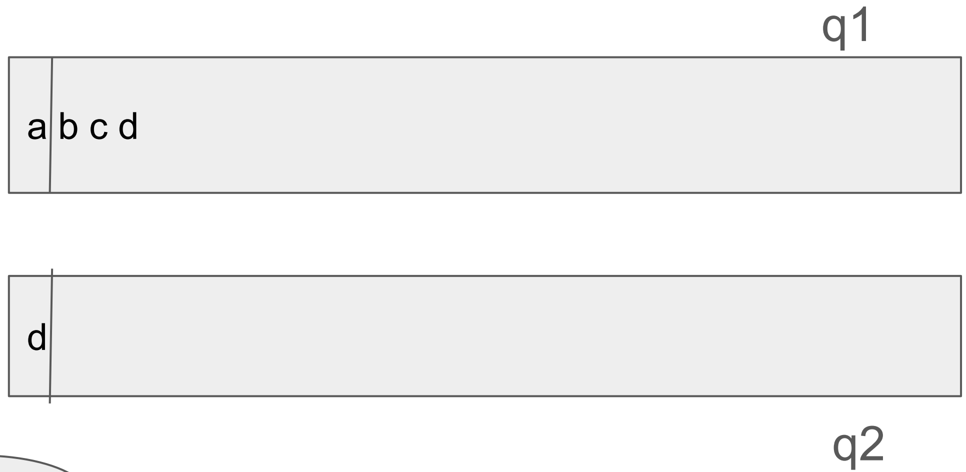

Push(a)
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Push(c)
Push(d)
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Pop() # should pop c

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Push(x):
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    if q2.size > 0: q2.deq()
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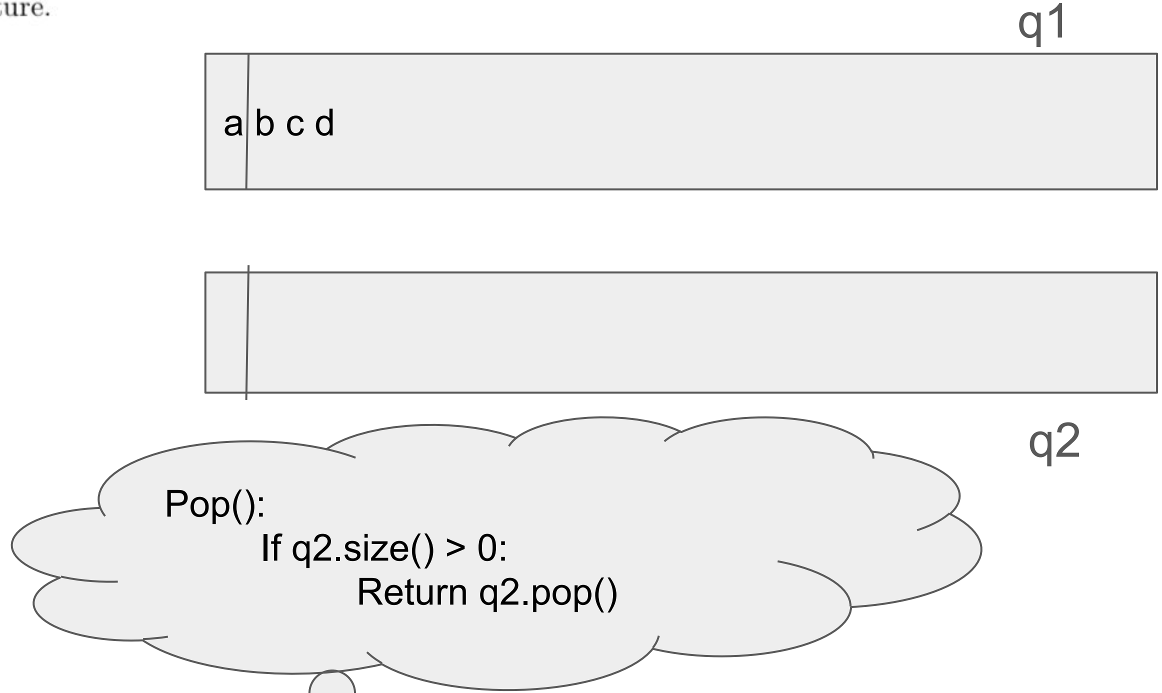

Last requirement: How do we get **c**?

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Push(b)
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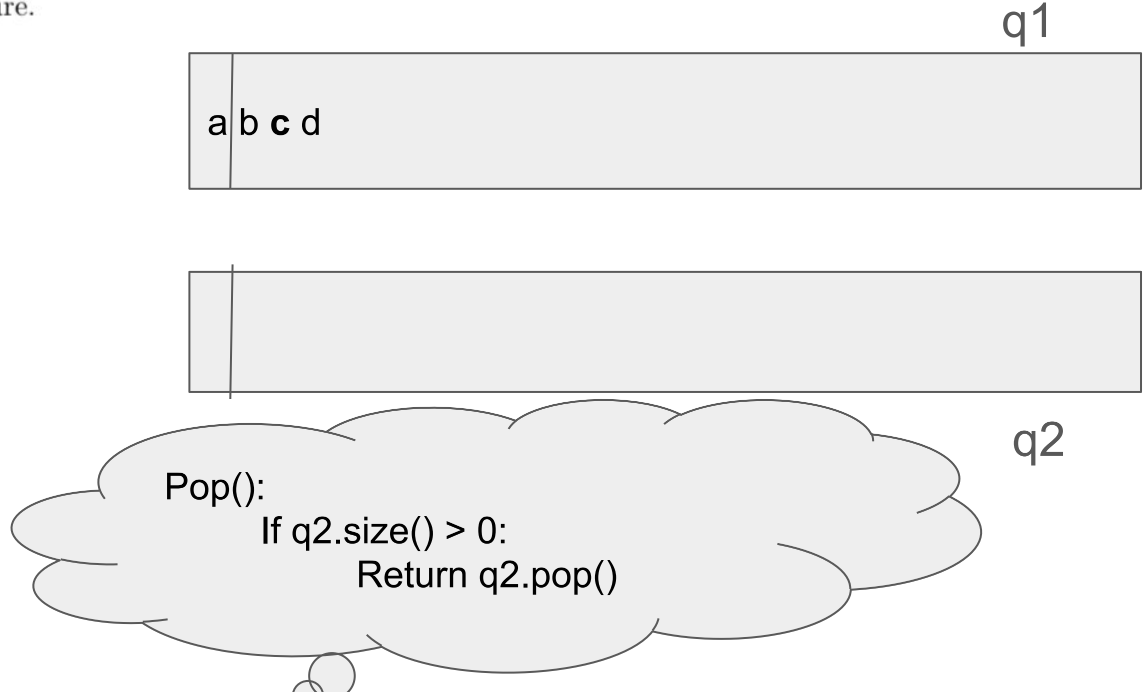

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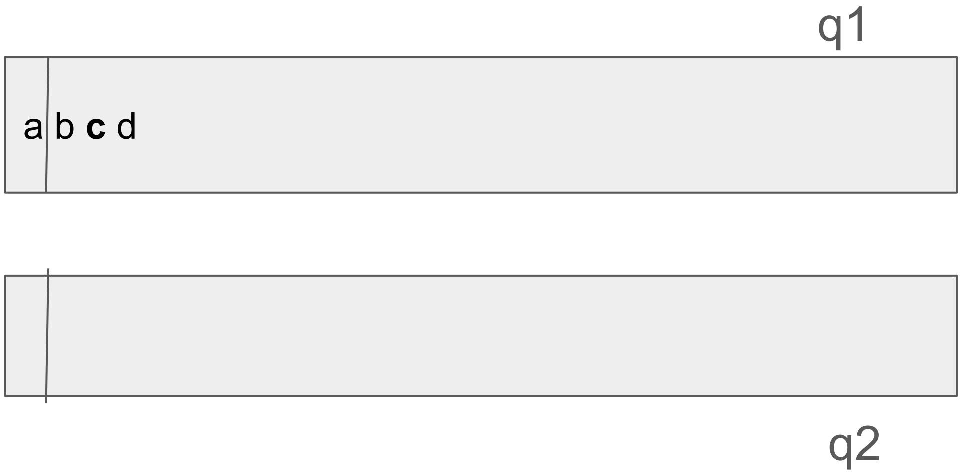

Push(a)
Push(b)
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Idea: Deque everything from q1 into q2
Keep track of elements seen to get c

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while q1.size > 0:
        seen = q1.pop()
        q2.enq(seen)
        #how to get c?
```

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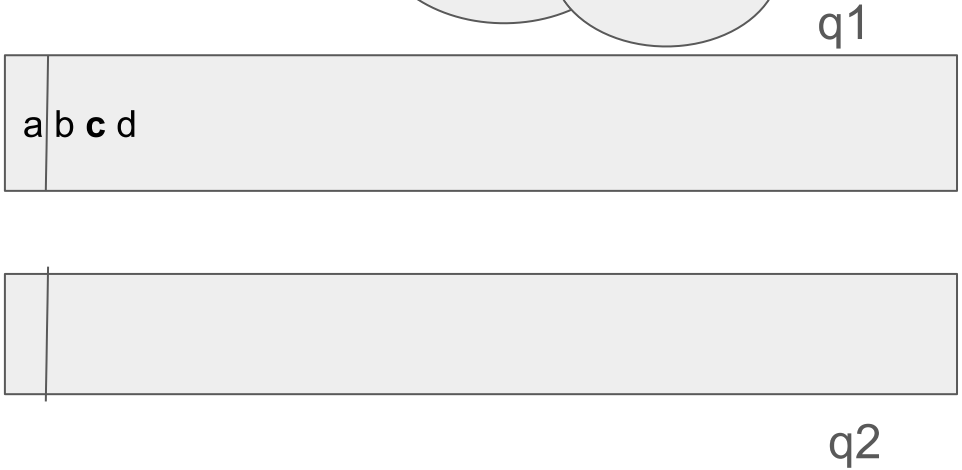

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```
while q1.size > 0:
        seen = q1.pop()
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        If q1.size() == 1:
            res = seen
```

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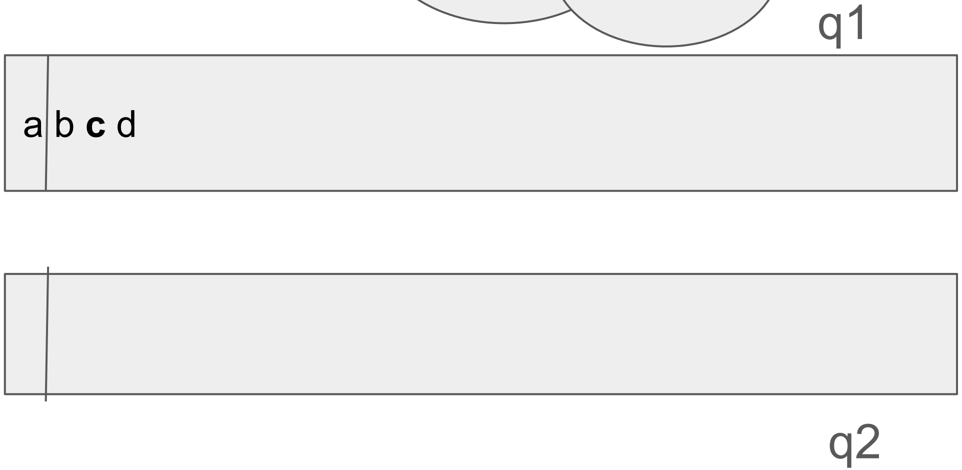

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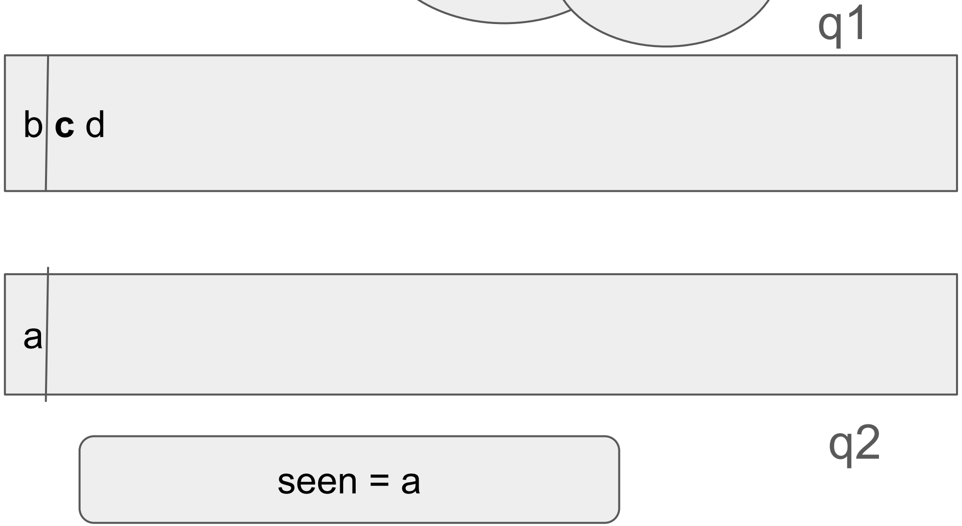

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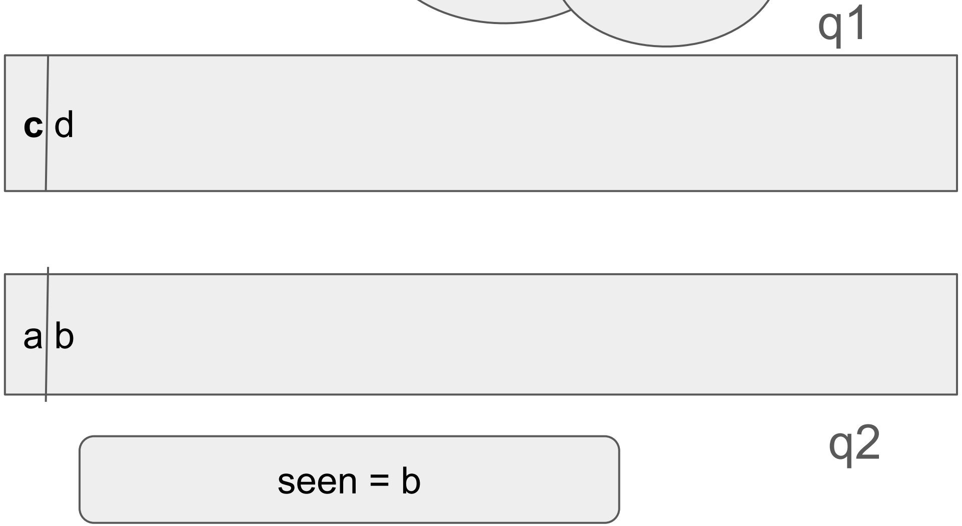

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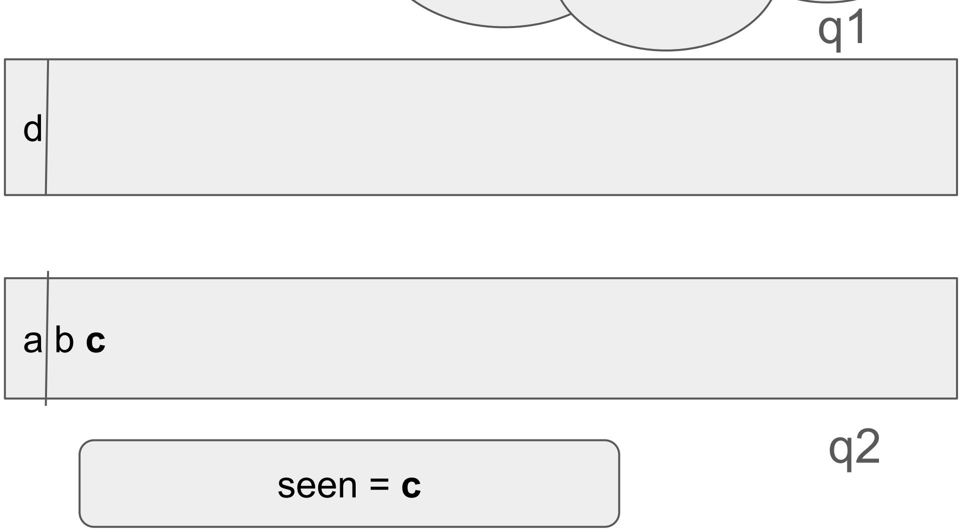
Push(c)
Push(d)
Pop() #should pop d
Pop() # should pop c



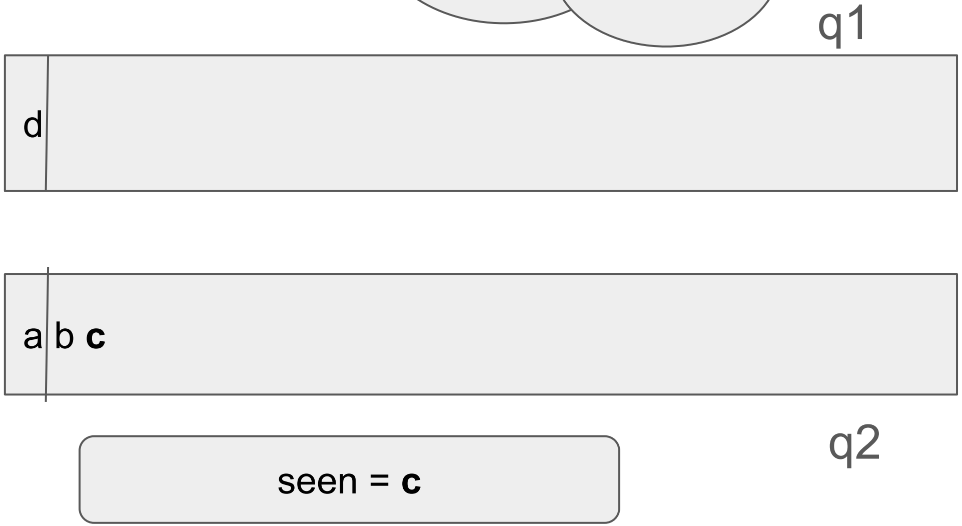
Idea: Deque everything from q1 into q2
Keep track of elements seen to get c

Last requirement: How do we get **c**?

1. Pushing an element to the stack takes no more than $O(1)$ operations.
2. Popping from the stack takes no more than $O(1)$ operations if performed after a push.
3. Popping from the stack takes no more than $O(n)$ operations if performed after another pop, n is the number of elements in the data structure.

```
while q1.size > 0:
        seen = q1.pop()
        q2.enq(seen)
        If q1.size() == 1:
            res = seen
```

Push(a)
Push(b)
Push(c)
Push(d)
Pop() #should pop d
Pop() # should pop c



Cool we have our result!
But our “stack” is ugly now.. How do we push/pop again?

Philosophy of Data Structures: Culling Chaos

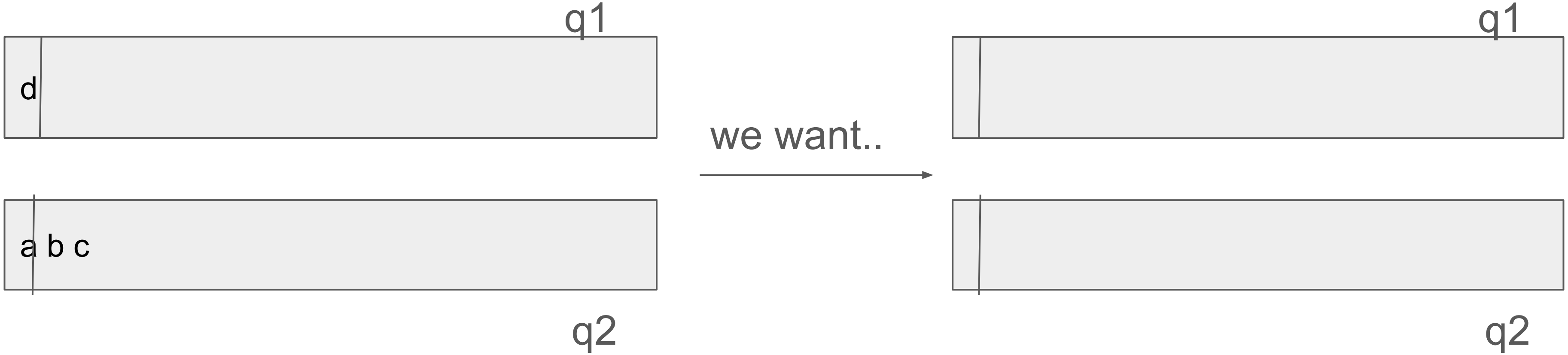
Sure fire design philosophy of data structures is **maintaining Invariants**

If I can make sure my data structures always look the same then easy to…

- Satisfy time efficiencies
- Write elegant pseudocode
- Prove/guarantee your impl. is efficient/correct

Example?

Invariant for our stack? After pop pop

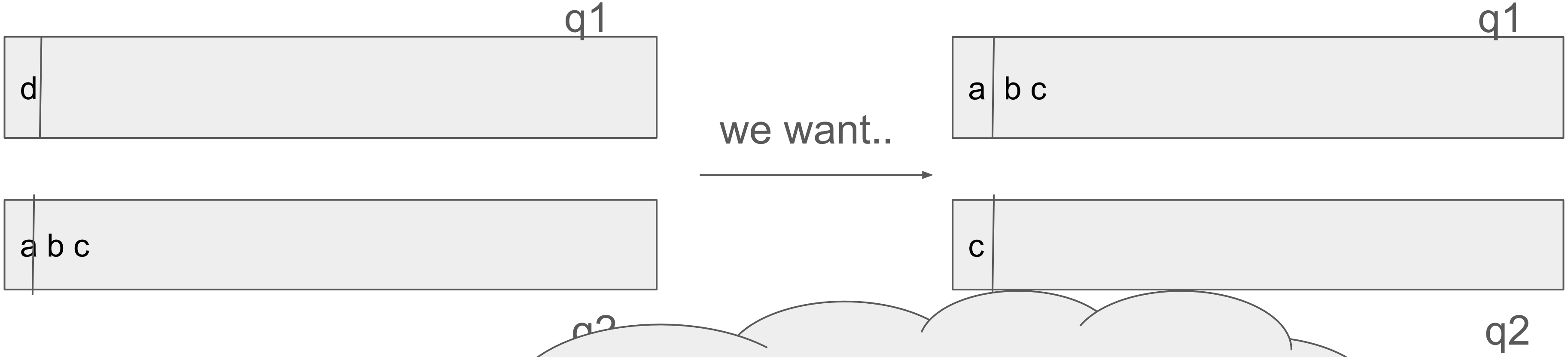


seen = c

Push(x):
    q1.enq(x)
    if q2.size > 0: q2.deq()
    q2.enq(x)

PushPop():
    If q2.size() > 0:
        Return q2.pop()

Invariant for our stack? After pop pop



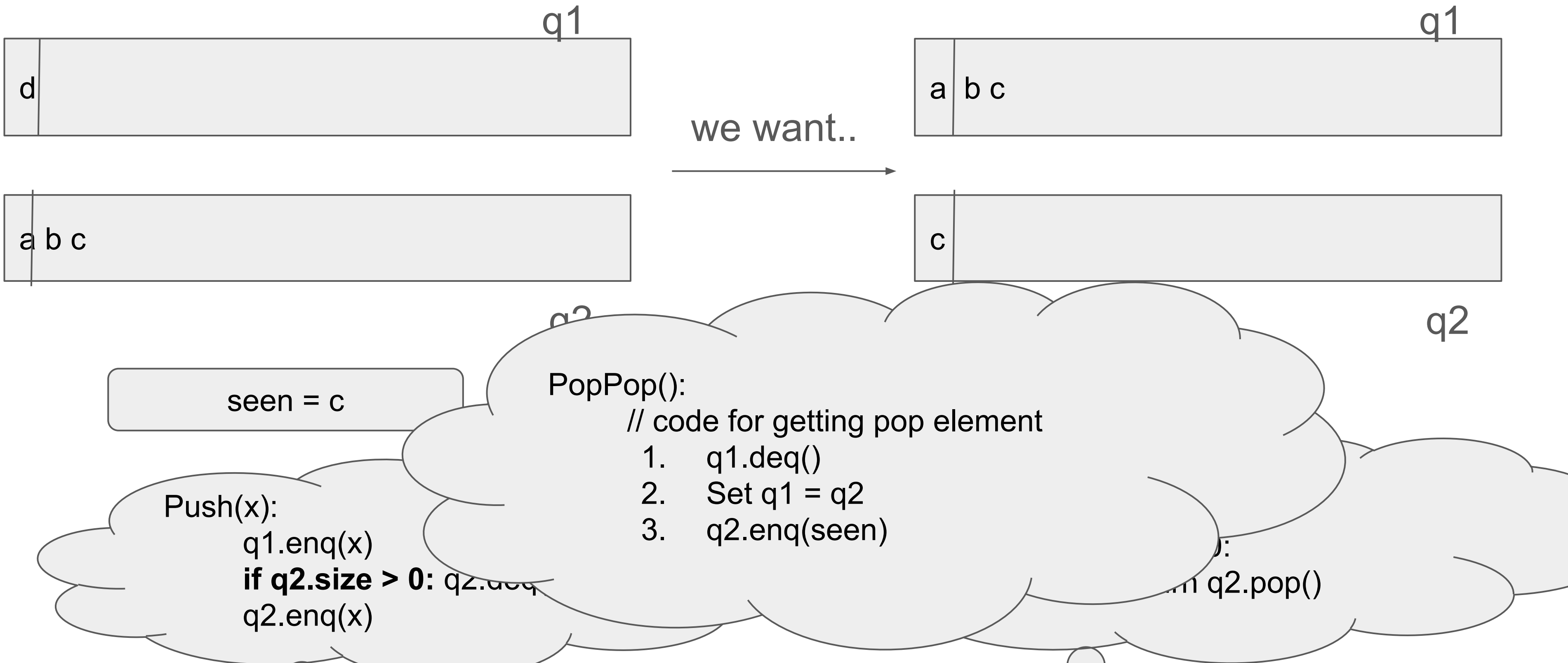
seen = c

```
PopPop():
        // code for getting pop element
        // TODO: code for fixing stack
```

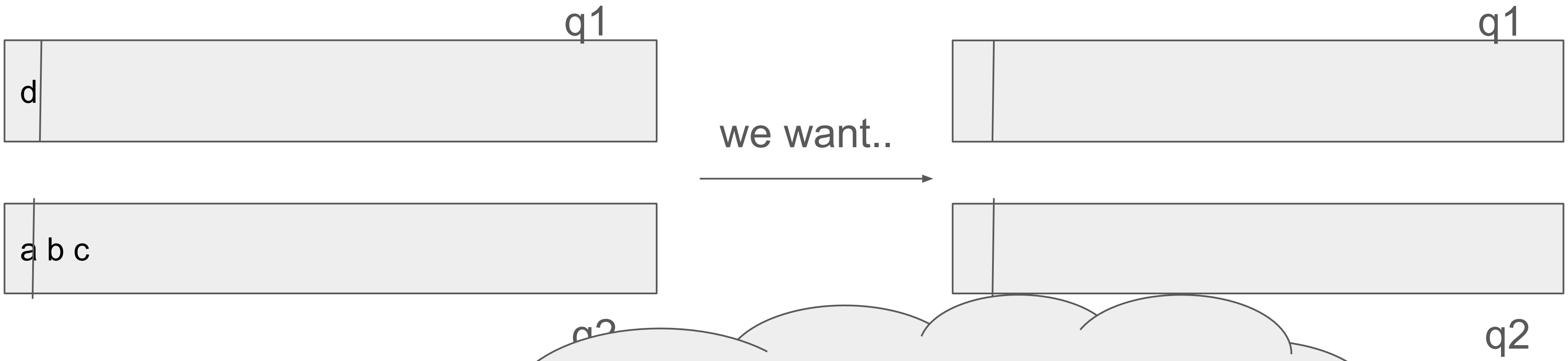
Push(x):
        q1.enq(x)
        if q2.size > 0: q2.deq
        q2.enq(x)

):
n q2.pop()

Invariant for our stack? After pop pop



Invariant for our stack? After pop pop



seen = c

PopPop():

// code for getting pop element

1. q1.deq()
2. Set q1 = q2
3. q2.enq(seen)

Push(x):

q1.enq(x)

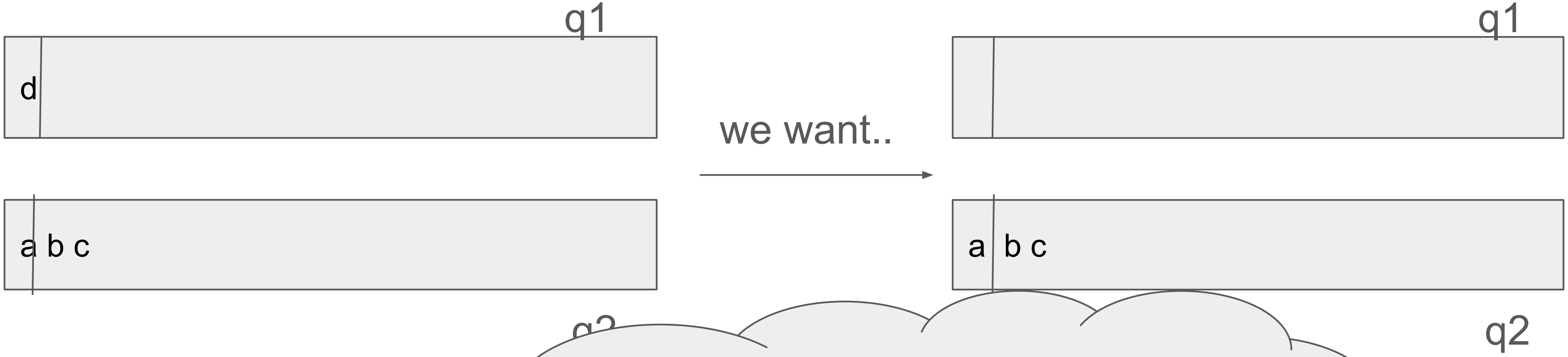
if q2.size > 0: q2.deq

q2.enq(x)

):

n q2.pop()

Invariant for our stack? After pop pop



seen = c

PopPop():

// code for getting pop element

1. **q1.deq()**
2. Set q1 = q2
3. q2.enq(seen)

Push(x):

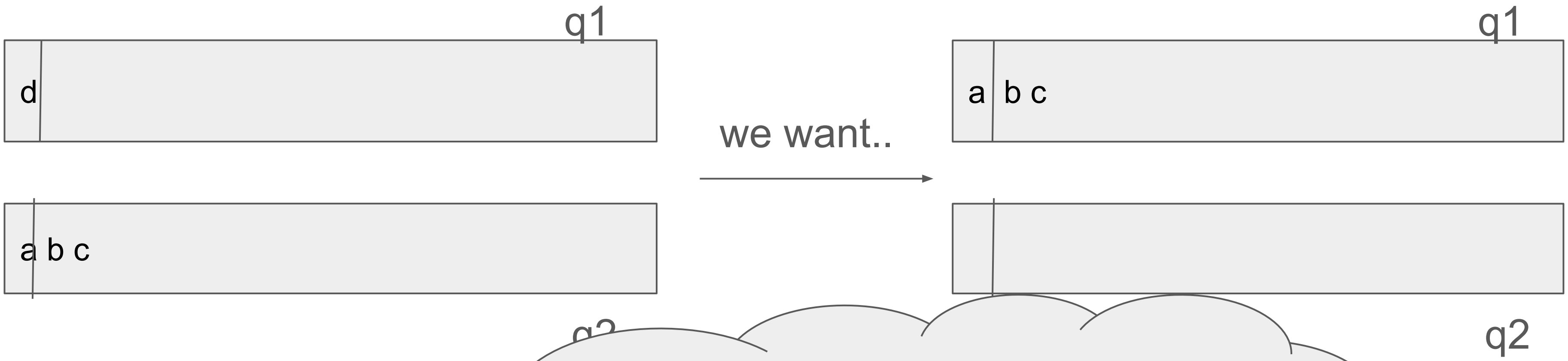
q1.enq(x)

if q2.size > 0: q2.deq

q2.enq(x)

q2.pop()

Invariant for our stack? After pop pop



seen = c

PopPop():

// code for getting pop element

1. q1.deq()
2. **Set q1 = q2**
3. q2.enq(seen)

Push(x):

q1.enq(x)

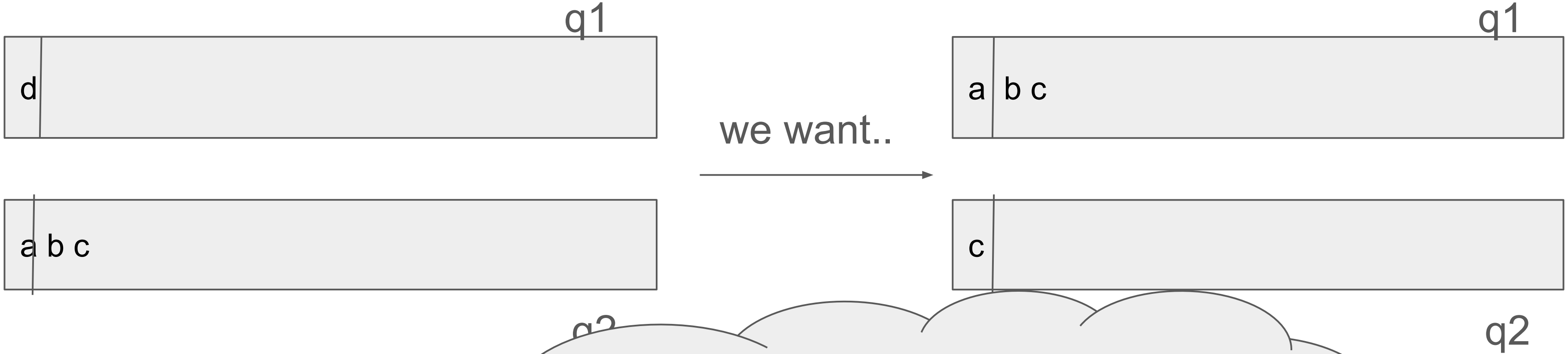
if q2.size > 0: q2.deq

q2.enq(x)

):

q2.pop()

Invariant for our stack? After pop pop



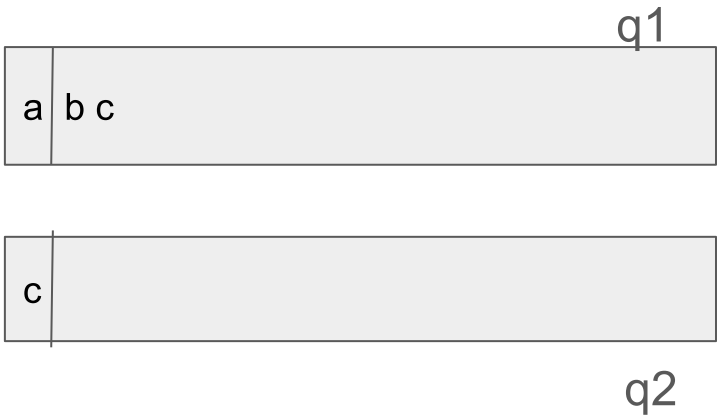
seen = c

PopPop():
  // code for getting pop element
  1. q1.deq()
  2. Set q1 = q2
  3. **q2.enq(seen)**

Push(x):
  q1.enq(x)
  if q2.size > 0: q2.deq
  q2.enq(x)

):
n q2.pop()

We don’t have to change our previous push/pop impl.!



seen = c

```
Push(x):
        q1.enq(x)
        if q2.size > 0: q2.deq()
        q2.enq(x)
```

```
PushPop():
        If q2.size() > 0:
                Return q2.pop()
```

Question 4

(Review)

(1) The big-O closed-form runtime expression $T(n)$ for the recurrence $T(n) = 3T(n/3) + n$ is (assume n is a power of 3 and $T(1) = 1$)

A. $O(n)$

B. $O(n \log n)$

C. $O(n^3 \log n)$

D. $O(\sqrt[3]{n} \log n)$

E. $O(n \sqrt[3]{\log n})$

(2) Two algorithms are developed based on the following template

```
function A(n : Z≥1 power of 2)
    if n = 1 then
        return 1
    end if
    ______________
    return A(n/2) + A(n/2)
end function
```

The missing part requires $F(n)$ time in Algorithm $\mathcal{A}_1$, and requires $G(n)$ time in Algorithm $\mathcal{A}_2$, where $F(n)$ and $G(n)$ are two functions of n.

If $F(n) = \Theta(G(n))$**, then** $\mathcal{A}_1(n) = \Theta(\mathcal{A}_2(n))$**.**

The above statement is

A. True

B. False

C. Possibly true/ Possible false

(3) Consider a sorted circular doubly-linked list where the head element points to the smallest element in the list. What is the time complexity to find the largest element in the list?

A. $O(1)$

B. $O(\log n)$

C. $O(n)$

D. $O(n \log n)$

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Same as T(n) = 2T(n / 2) + n,
Solve using tree method.

Exercise

For constant k, show T(n) = kT(n/ k) + n is O(nlgn)

(2) Two algorithms are developed based on the following template

```
function A(n : Z≥1 power of 2)
    if n = 1 then
        return 1
    end if
    ____________
    return A(n/2) + A(n/2)
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The above statement is

A. True

B. False

C. Possibly true/ Possible false

```
function A(n : Z≥1 power of 2)
    if n = 1 then
        return 1
    end if
    ______F or G(n)
    return A(n/2) + A(n/2)
end function
```

If $F(n) = \Theta(G(n))$**, then** $\mathcal{A}_1(n) = \Theta(\mathcal{A}_2(n))$**.**

Tree method:

Recurrence:

Tree:

Cost per level i:

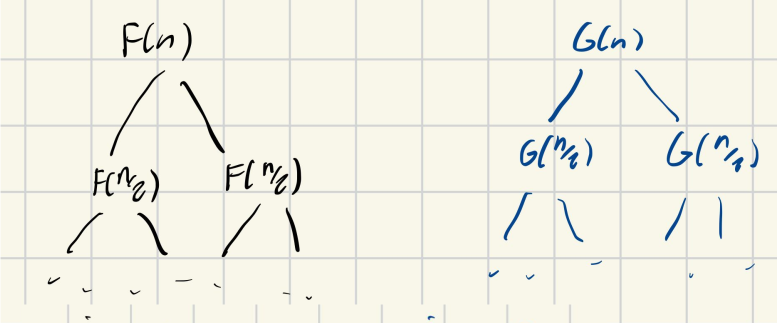
Levels:

Sum

```
function A(n : Z≥1 power of 2)
    if n = 1 then
        return 1
    end if
    ________ F or G(n)
    return A(n/2) + A(n/2)
end function
```

If $F(n) = \Theta(G(n))$, then $\mathcal{A}_1(n) = \Theta(\mathcal{A}_2(n))$.

Tree method:



wts:

Cost per level i: $2^i F(n/2^i)$ $\quad$ $2^i G(n/2^i)$

Levels: lg n

Sum:

$$\sum_{i=1}^{\lg n} 2^i F(n/2^i) \qquad \sum_{i=1}^{\lg n} 2^i G(n/2^i)$$

If $F(n) = \Theta(G(n))$, then $\mathcal{A}_1(n) = \Theta(\mathcal{A}_2(n))$.

$$\sum_{i=1}^{\lg n} 2^i F(n/2^i) \qquad \sum_{i=1}^{\lg n} 2^i G(n/2^i)$$

wts:

$$\sum_{i=1}^{\lg n} 2^i F(n/2^i) \in \Theta\left(\sum_{i=1}^{\lg n} 2^i G(n/2^i)\right)$$

This is true if ..

WTS:

$$\sum_{i=1}^{\lg n} 2^i F(n/2^i) \in \Theta\left(\sum_{i=1}^{\lg n} 2^i G(n/2^i)\right)$$

This is true if for each $i = 1, 2, \ldots, \lg n$

$$2^i F(n/2^i) \in \Theta\left(2^i G(n/2^i)\right)$$

And this is true if..

wts:

$$\sum_{i=1}^{\lg n} 2^i F(n/2^i) \in \Theta\left(\sum_{i=1}^{\lg n} 2^i G(n/2^i)\right)$$

This is true if for each $i = 1, 2, \dots, \lg n$

$$2^i F(n/2^i) \in \Theta(2^i G(n/2^i))$$

And this is true if

$$F(n/2^i) \in \Theta(G(n/2^i))$$

Which is true by our if condition!

If $F(n) = \Theta(G(n))$, **then** $\mathcal{A}_1(n) = \Theta(\mathcal{A}_2(n))$.

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